

## MATH 302 WEEK 7 HOMEWORK

Consider the Levins metapopulation model.<sup>1</sup> A metapopulation is a population of populations which are spatially separated and so interact in a limited manner. (Think, for example, humans spread across the globe, or a species of fish which lives in different regions in the ocean.) Richard Levins's model breaks the metapopulation up into possible patches which can be inhabited, with high interaction within a patch and low interaction between patches. He makes the simplifying assumption that each patch is either populated or empty, with no shades of gradation between.

Levins's model makes the following assumptions about the patches: Let  $P$  be the fraction of patches occupied at a given time. During an interval of time  $dt$  each occupied patch becomes extinct with probability  $\varepsilon dt$ . And each empty patch become populated proportional to the number of populated patches and some constant. So an empty patch becomes filled with probability  $\varphi P dt$ , for a constant  $\varphi$ .

(a) Using the above assumptions, write a differential equation which describes  $dP/dt$  in terms of the fraction  $P$  of patches occupied, the filling constant  $\varphi$ , and the extinction constant  $\varepsilon$ . (Hint: first use the assumptions to write  $dP$ , the change in  $P$ , in terms of  $dt$ .)

(b) Solve this differential equation to express  $P$  as a function of  $t$ , based upon the constants  $\varphi$  and  $\varepsilon$ , and a new constant  $K$  arising from integrating. (You may use a computer algebra system to help with this, but clearly set up the integral(s) to be solved and indicate where you used the CAS to help you.)

(c) A metapopulation of sharks occupies a number of patches in the Pacific. Empirical data gathering has found that the extinction constant  $\varepsilon$  is  $1/10$  and the filling constant  $\varphi$  is  $1/5$ . Empirical data gathering also found that in the year 2000 the shark population filled  $1/10$  of the patches. Assuming the shark metapopulation fits the Levins model, estimate what percentage of the patches will be filled in 2020. (Hint: Use 2000 as time  $t = 0$ .)

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<sup>1</sup>Richard Levins, "Some demographic and genetic consequences of environmental heterogeneity for biological control", *Bulletin of the Entomological Society of America*, Vol. 15, pp. 237—240 (1969).