MAKE UP FOR EXAM 2, MATH 244

There are six questions here, corresponding to each problem on the in-class portion of the midterm. Choose up to three questions on the midterm you lost points for. Do the corresponding problems here, and turn them into me on Monday, April 22, along with returning the in-class portion of your exam. This will net you up to 70% of the points you lost (rounded down to the nearest point) on those three problems.

(1) Use Green's theorem to compute the circulation of $\vec{F} = (xy, x^2 + y^2)$ along the unit circle centered at the origin, oriented counterclockwise.

(2) Let C be the portion of the parabola $y = x^2 - 1$ from the point $(-1, 0)$ to $(1, 0)$ and let $\vec{F} = (4, xy)$. Compute the flow of $\vec{F}$ along C, going in the direction of increasing x. And compute the flux of $\vec{F}$ across C, oriented in the direction of decreasing y.

(3) Let $\vec{F} = (x^2y, 2y^2z, 3xz^2)$. Compute $\operatorname{grad}\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$.

(4) Consider $\vec{F} = (ye^z - y + 2x, xe^z - x, xye^z + 2)$.

   (a) Show that $\vec{F}$ is conservative.

   (b) Find a potential function for $\vec{F}$.

   (c) Compute the flow of $\vec{F}$ along the following curves:

      (i) A regular heptakaidecagon (17-gon), oriented clockwise.

      (ii) The portion of the circle of radius $\sqrt{2}$ in the xy-plane centered at the origin from $(-1, 1, 0)$ to $(1, 1, 0)$.

(5) Let S be the portion of the cone $z = 2\sqrt{x^2 + y^2}$ which lies between the planes $z = 2$ and $z = 8$. Find a parameterization for S, along with its domain. Use this parameterization to calculate the surface area of S.

(6) Let C be the curve parameterized by $\vec{r}(t) = (t, t^2, t^3)$ with $1 \le t \le 2$ oriented in the direction of increasing t. Calculate the following path integral

$$\int_C 8x + 36z \,\mathrm{d}s.$$

Date: April 18, 2019.