Study guide for Math 244 Midterm 2

April 4, 2019

These are the sorts of questions you should know how to solve for the first midterm.

1. Let $\vec{F} = xy\vec{\imath} + e^{yz}\vec{\jmath} + (2y+z)\vec{k}$. Calculate $\operatorname{curl}\vec{F}$ and $\operatorname{div}\vec{F}$.

2. Calculate the surface area of the surface $z = x^2 + y$ with $-1 \le x \le 1$ and $0 \le y \le 2$.

3. Calculate $\int_C \vec{F} \dot{\mathrm{d}}\vec{r}$ where C is the portion of the unit circle from the point $(1,0)$ to $(0,1)$ and $\vec{F} = (x^2, -y)$.

4. Use Green's theorem to calculate the counterclockwise circulation of $\vec{F} = (ye^x, xe^y)$ around the square with corners $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$.

5. Let $\vec{F} = (M, N, P)$ be a vector field whose components all have continuous second partial derivatives. Show that $\operatorname{div}\operatorname{curl}\vec{F} = 0$.

6. The *Laplacian* of a scalar-valued function $f(x,y)$ is $\nabla \cdot \nabla f$. A function $f(x,y)$ is *harmonic* if its Laplacian is 0. Show that $f(x,y) = \log(x^2+y^2)$ is harmonic.

7. Consider the curve C parameterized by $\vec{r}(t) = (t, t^2, e^t)$ with $0 \le t \le 100$ and let $\vec{F} = (xy, -y/2, 1/z)$. Calculate
$$\int_C \vec{F} \cdot \mathrm{d}\vec{r}.$$

8. Check that the vector field $\vec{F} = yz\cos(xy)\vec{\imath} + xz\cos(xy)\vec{\jmath} + \sin(xy)\vec{k}$ is conservative. Find a potential function for $\vec{F}$. Calculate the path integral
$$\int_{(0,0,0)}^{(1,\pi,1)} \vec{F} \cdot \vec{T}\,\mathrm{d}s.$$

9. Compute the integral
$$\int_C \vec{F} \cdot \mathrm{d}\vec{r}$$
where $\vec{F} = y\vec{\imath} + x\vec{\jmath}$ and C is the curve parameterized by $\vec{r}(t) = t^7\vec{\imath} + t^{11}\vec{\jmath}$ with $0 \le t \le 1$.

10. Use the coordinate transform $x = u^2$, $y = v$ to compute the integral
$$\int_0^{\sqrt{15}} \int_0^{\sqrt{x}} 4y\sqrt{x^2+1}\,\mathrm{d}y\,\mathrm{d}x.$$

11. Show that
$$\int_C \left((e^x \sin y + 3x^2)\vec{\imath} + (e^x \cos y + 4y^3)\vec{\jmath}\right) \cdot \mathrm{d}\vec{r} = 0$$
where C is the counterclockwise-oriented regular heptakaidecagon (17-gon) centered at the origin with one vertex on the point $(1,0)$. Explain in words why your calculations show that this integral is 0.

You are not expected to memorize Green's theorem.