MATH455 HOMEWORK 5
DUE FRIDAY, FEBRUARY 28

This homework set references the sequent rules sheet I handed out in class, which also can be found linked from the course website. Recall that a sequent rule of the form

$$\frac{\Gamma_0 \vdash \varphi_0 \qquad \Gamma_1 \vdash \varphi_1}{\Gamma \vdash \varphi}$$

is *sound* if $\Gamma_0 \models \varphi_0$ and $\Gamma_1 \models \varphi_1$ implies $\Gamma \models \varphi$. (And similarly for rules with only one input sequent.)

Exercise 1. Argue that the Contrapositive sequent rule is sound.

Exercise 2. Argue that the two $\forall$ sequent rules are sound.

Exercise 3. Do Exercise 5.5 (a1) and (a2) from the textbook (page 69). You can use all of the sequent rules from the sheet I gave.

Exercise 4. Consider the following sequent rule:

$$\frac{\Gamma \vdash \varphi \Rightarrow \psi}{\Gamma, \varphi \vdash \psi}$$

Show that this rule can be derived from the sequent rules on the sheet I gave you. Conclude that if $\Gamma \vdash \varphi \Rightarrow \psi$ (that is, Γ syntactically entails $\varphi \Rightarrow \psi$) then $\Gamma \cup \{\varphi\} \vdash \psi$ (that is, $\Gamma \cup \{\varphi\}$ syntactically entails ψ).