

# Study guide for Math 1410 Final Exam

November 24, 2021

For the final exam, you should know the topics from the first two midterms (see the associated study guides), as well as the following topics.

- (a) How to use trig functions and inverse trig functions to determine angles and side lengths of right triangles, given partial information about a right triangle.
- (b) How to simplify expressions involving trig functions with inverse trig functions as inputs.
- (c) How to use trig identities to compute values for sine and cosine. (I won't ask you about the tangent identities.)
- (d) How to use trig identities to simplify trigonometric expressions.

Note that I will not ask you to memorize trig identities (besides the Pythagorean identity and the definitions of the other trig functions in terms of sine and cosine). If an exam question needs a trig identity, I will give it to you with the question.

These are the sorts of questions you should know how to solve for the final.

- (1) A right triangle has an angle of  $40^\circ$  and the side opposite that angle has length 6. What are the lengths of the other two sides? (Round your answer to two decimal points.)
- (2) Simplify  $\tan(\arcsin(1+x))$ .
- (3) Use the angle-sum identity to calculate  $\sin(-\pi/12)$ .
- (4) Suppose the angle  $\alpha$  is in Quadrant IV and the angle  $\beta$  is in Quadrant II. If  $\sin \alpha = -2/7$  and  $\sin \beta = 1/7$ , determine  $\sin^2(\alpha + \beta)$ .
- (5) Suppose the angle  $\alpha$  is in Quadrant I and  $\cos \alpha = 4/9$ . What is  $\cos(2\alpha) + \sin(2\alpha)$ ?
- (6) Show that the expression  $\cos(2x)$  simplifies to  $(\cos x - \sin x)(\cos x + \sin x)$ .
- (7) Sketch a graph of  $f(x) = 2\cos(2x - \pi)$ , showing one full period. What is its period and amplitude?
- (8) Sketch a graph of  $g(x) = 3 \cdot 2^{x-1}$ , identifying all asymptotes. What are the domain and range of  $g(x)$ ?
- (9) Sketch a graph of  $h(x) = \log_3((2x-3)^2)$ , identifying all asymptotes. What are the domain and range of  $h(x)$ ?
- (10) Sketch a graph of  $j(x) = 3(x-2)^3(x+2)^2(x-4)(x+4)^3$ , identifying all roots and where  $j(x)$  is positive/negative.
- (11) Sketch a graph of  $k(x) = 2x^2 - x - 3$ , identifying its vertex and all roots. What are the domain and range of  $k(x)$ ?
- (12) Sketch a graph of  $\ell(x) = \frac{2x-1}{x+2}$ , identifying all asymptotes. What are the domain and range of  $\ell(x)$ ?
- (13) Find all points where the graphs of  $f(t) = t^2 - 3t + 2$  and  $g(t) = -t^2 + 9$  intersect.

- (14) Find the preimage of the interval  $(0, 3]$  under the function  $a(x) = 2^{x-1}$ .
- (15) Find the image of the interval  $[-2, 2]$  under the function  $b(x) = x^2 - 4$ .
- (16) What is the difference quotient of  $f(x) = x^2 \log_4((x-2)(x+2))$  from  $x = 2$  to  $x = 4$ ?
- (17) Consider the function

$$F(y) = \frac{x^2}{\sqrt{x^3 - 4x}}.$$

What is the domain of  $F(y)$ ? Express your answer in interval notation.

If you want more to study from, I advise looking over the two midterms and the sample problems from their study guides.