Unit 1 Assessment D

Learning Objective	Grade
Rates of Change	
Pointwise Behavior	
Global Behavior	
Graphing	
Rewriting Equations	
Function Algebra	

Each graded out of 100, each graded separately.

Functions as Quantities Changing with Each Other

(40 pts) 1. The function $f(x)$ is graphed below:
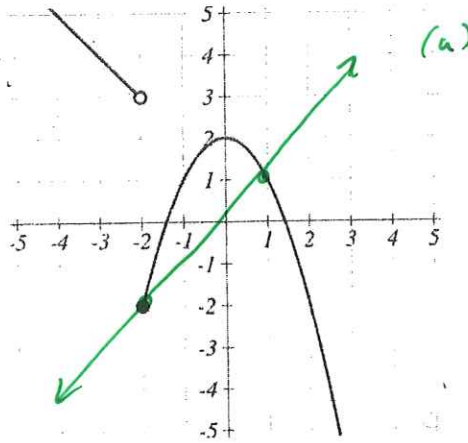
(a) Draw a line that is secant to f at $x = -2$ and $x = 1$.

(b) What is the average rate of change of f between $x = -2$ and $x = 1$.

$$ARC = \frac{f(1) - f(-2)}{1 - (-2)} = \frac{1 - (-2)}{3} = \frac{3}{3} = \boxed{1}$$

(c) Describe how f is changing over $(-\infty, -2)$.

- ○ increasing at a constant rate
- ○ increasing at an increasing rate
- ○ increasing at a decreasing rate
- ✓ decreasing at a constant rate
- ○ decreasing at an increasing rate
- ○ decreasing at a decreasing rate

(d) Describe how f is changing over $(-2, 0)$.

- ○ increasing at a constant rate
- ○ increasing at an increasing rate
- ✓ increasing at a decreasing rate
- ○ decreasing at a constant rate
- ○ decreasing at an increasing rate
- ○ decreasing at a decreasing rate

(30 pts) 2. The continuous function $m(t)$ is represented in the table below:

t	0	1	2	3	4	5	6	7
$m(t)$	−1	1	4	9	11	13	15	17

0   2 3 5   2 2 2

(a) What is the average rate of change of m over $[0, 4]$?

$$ARC = \frac{m(4) - m(0)}{4 - 0} = \frac{11 - (-1)}{4} = \frac{12}{4} = 3$$

(b) Describe how $m(t)$ is changing over $(0, 3)$.

- ○ increasing at a constant rate
- ✓ increasing at an increasing rate
- ○ increasing at a decreasing rate
- ○ decreasing at a constant rate
- ○ decreasing at an increasing rate
- ○ decreasing at a decreasing rate

(c) Describe how $m(t)$ is changing over $(4, 7)$.

- ✓ increasing at a constant rate
- ○ increasing at an increasing rate
- ○ increasing at a decreasing rate
- ○ decreasing at a constant rate
- ○ decreasing at an increasing rate
- ○ decreasing at a decreasing rate

(30pts) 3. Let $f(x) = x^2 - 9$. Find the average rate of change of f over $[x, x+h]$. You must show a step-by-step solution to receive credit.
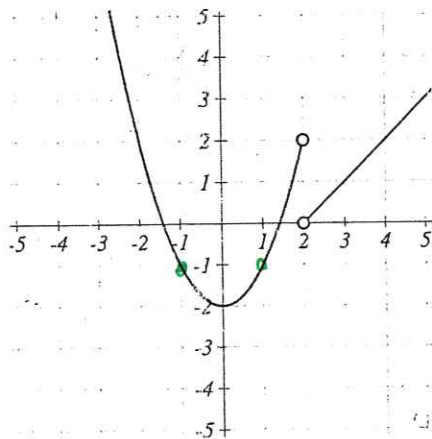
$$ARC = \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 9 - (x^2 - 9)}{h}$$

$$= \frac{x^2 + 2hx + h^2 - 9 - x^2 + 9}{h} = \frac{2hx + h^2}{h}$$

$$\boxed{= 2x + h}$$

Behavior of Function at a Point

(40 pts)

4. The graph of k is given below:

Find the requested values. If any of them are not integer values, you may give approximate decimal answers.

(a) The value of $k(2)$

Undefined /DNE

(b) The value of c if $k(c) = -1$

$c = -1$ or 1

(c) The zeros of k

$-1.5, 1.5$

(d) The initial value of k

-2

(30 pts)

5. Let $w(x) = \frac{mx}{n} + 3n^2$ for constants $m, n \in \mathbb{R}$. Find each of the following:

(a) The x-intercept(s) of w.

$$0 = \frac{mx}{n} + 3n^2$$

$$\frac{mx}{n} = -3n^2$$

$$mx = -3n^3$$

$$\boxed{x = \frac{-3n^3}{m}}$$

x-int $\Rightarrow$ $\left(-3n^3/m, 0\right)$

(b) The initial value(s) of w.

$$w(0) = 0 + 3n^2$$

$$= 3n^2$$

(3pts)

6. Find the point(s) on the graph of f with a y-value of -3 if $f(x) = x^2 - 2x - 5$.

$$x^2 - 2x - 5 = -3$$

$$x^2 - 2x - 2 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-2)}}{2}$$

$$= \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$$
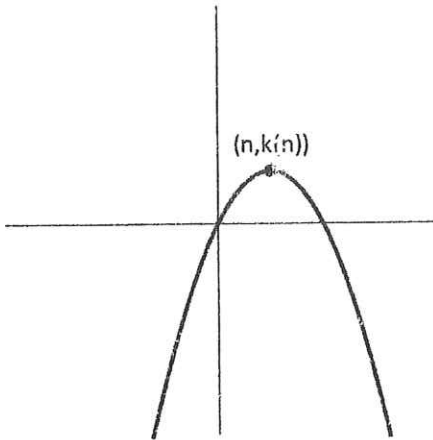
$$= 1 \pm \sqrt{3}$$

p-nts: $\left(1 + \sqrt{3}, -3\right), \left(1 - \sqrt{3}, 3\right)$

Behavior of Function Over an Interval

(25pts)

7. The function $k(x)$ is graphed below:

(n,k(n))

Find each of the items to the right for this function.

(a) The increasing intervals of k.

$(-\infty, n)$

(b) The decreasing intervals of k.

(n, ∞)

(c) What is the domain of k?

~~XXXXXXX~~ $(-\infty, \infty)$

(d) What is the range of k?

$(-\infty, k(n)]$

(e) Complete the following sentence:

The function k has an absolute minimum/*maximum* (circle one) of $k(n)$ at $x =$ n.

(35pts)

8. Let $j(x) = -2(x+1)^2 - 5$. State each of the following: Vertex at $(-1, -5)$

(a) The increasing intervals of j.

$(-\infty, -1)$

(b) The decreasing intervals of j.

$(-1, \infty)$

(c) Is g concave up or concave down?

○ concave up
◉ concave down
○ neither

(d) What is the domain of j?

$(-\infty, \infty)$

(e) What is the range of j?

$(-\infty, -5]$

(f) Fill in the blanks below:

$x \to -\infty, j(x) \to$ $-\infty$

$x \to \infty, j(x) \to$ $-\infty$

(g) Complete the following sentence:

The function j has an absolute minimum/*maximum* (circle one) of -5 at $x =$ -1.

(40pts)

9. Let $f(x) = x^2 - 2x + 8$. State each of the following. *Hint: Finding the vertex may help you answer the following questions.*

(a) The increasing intervals of f.

$$(1, \infty)$$

vertex at $(1, 7)$

$$x = -\frac{(-2)}{2 \cdot 1} = 1$$

(b) The decreasing intervals of f.

$$(-\infty, 1)$$

$$f(1) = 1^2 - 2 + 8$$
$$= 7$$

(c) Is f concave up or concave down?
- ● concave up
- ○ concave down
- ○ neither

(d) What is the domain of f?

$$(-\infty, \infty)$$

(e) What is the range of f?

$$[7, \infty)$$

(f) Fill in the blanks below:

$x \to -\infty, f(x) \to \underline{\quad \infty \quad}$

$x \to \infty, f(x) \to \underline{\quad \infty \quad}$

(g) Complete the following sentence:

The function f has an absolute *minimum*/*maximum* (circle one) of $\underline{\quad 7 \quad}$ at $x = \underline{\quad 1 \quad}$.

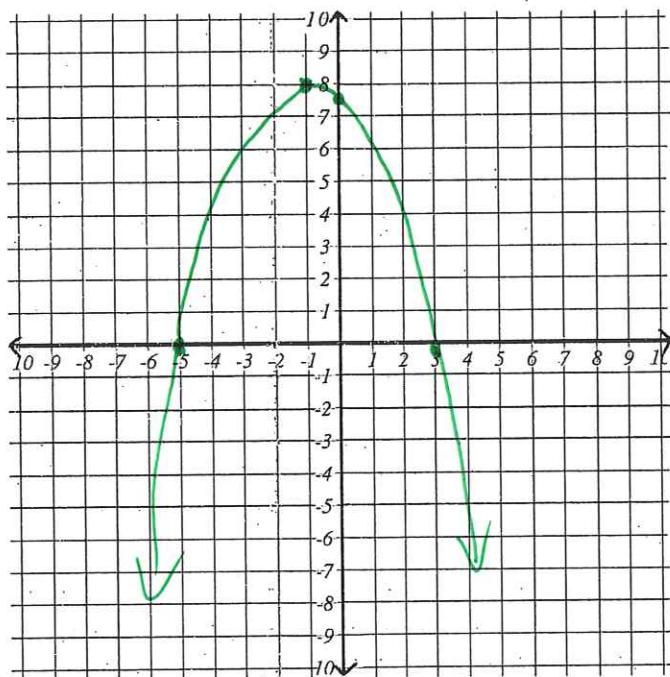**Graphs of Functions**

(30pts)

10. Graph the function $k(t) = \begin{cases} -6-t & t < -2 \\ -4 & -2 \leq t < 0 \\ 5 & t > 0 \end{cases}$.
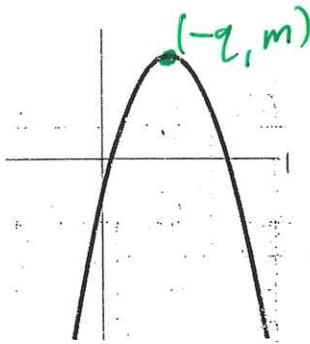
$-6-(-2) = -4$

$-6-(-3) = -3$

(25pts)
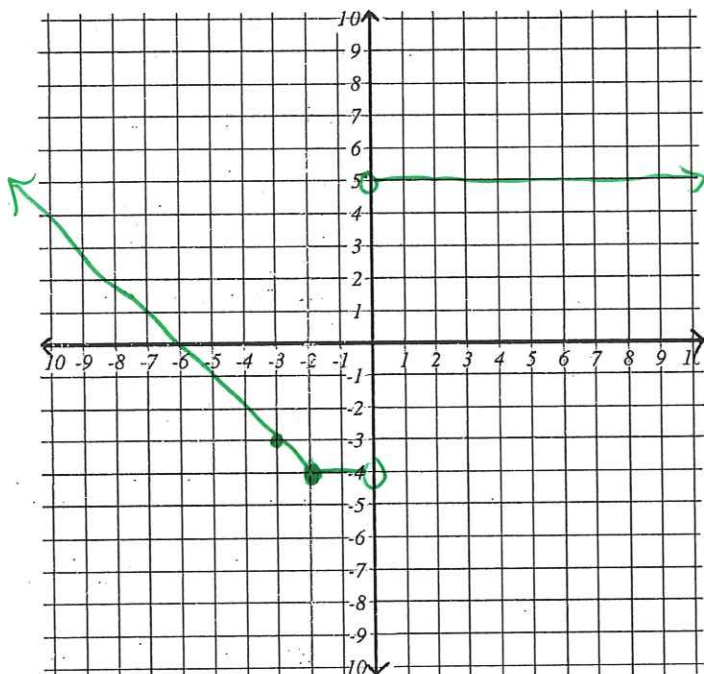
11. A quadratic function has zeros of -5 and 3. It also has an initial value of 7.5. This function has a maximum of 8 which occurs at $x = -1$. Sketch a graph of the function on the grid below by plotting its intercepts and vertex.

(45pts)

12. The following are graphs of the function $f(t) = m + n(t+q)^2$ with constants $m, n, q \in \mathbb{R}$ are given below. Determine which of the following statements below each graph are true about the values for each constants for the function graphed above.
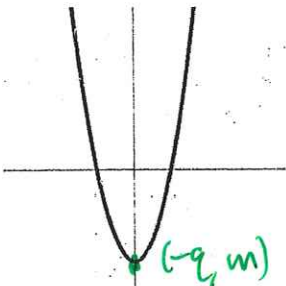
$(-q, m)$

- ○ $m < 0$
- ● $m = 0$ ❌
- ● $m > 0$

- ● $n < 0$
- ○ $n = 0$
- ○ $n > 0$

- ● $q < 0$
- ○ $q = 0$
- ○ $q > 0$

$(-q, m)$

- ● $m < 0$
- ○ $m = 0$
- ○ $m > 0$

- ○ $n < 0$
- ○ $n = 0$
- ● $n > 0$

- ○ $q < 0$
- ● $q = 0$
- ○ $q > 0$

$(-q, m)$

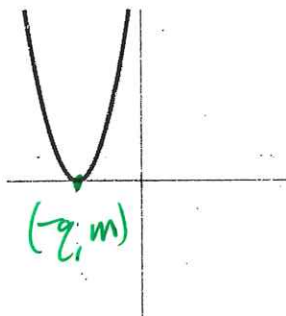- ○ $m < 0$
- ● $m = 0$
- ○ $m > 0$

- ○ $n < 0$
- ○ $n = 0$
- ● $n > 0$

- ○ $q < 0$
- ○ $q = 0$
- ● $q > 0$

Rewriting Equations of Functions

(40pts)

13. Complete the square to rewrite $h(x) = x^2 - 8x + 5$ in the form $y = a(x+h)^2 + k$.

$$h(x) = x^2 - 8x + \underbrace{16 - 16}_{} + 5$$

$$= (x-4)^2 - 16 + 5$$

$$h(x) = (x-4)^2 - 11$$

(60, ts)

14. Rewrite each of the following function's equations in factored form.

(a) $f(n) = 90n^2 - 10$

$$f(n) = 90\left(n^2 - \tfrac{1}{9}\right) = 90\left(n - \tfrac{1}{3}\right)\left(n + \tfrac{1}{3}\right)$$

$$\text{OR!} \quad f(n) = 10\left(9n^2 - 1\right) = 10(3n-1)(3n+1)$$

(b) $g(x) = x^2 - 8x + 12$

$$g(x) = (x-2)(x-6)$$

(c) $k(t) = 7t^2 + 8t + 1$

$$k(t) = (7t+1)(t+1)$$

$$= 7\left(t + \tfrac{1}{7}\right)(t+1)$$

(100 pts)

Function Algebra

15. Let $h(x) = 2x - 10$ and $j(x) = 5x^2$. Find each of the following and write the equation in simplified in descending order.

(a) $(j \circ h)(x)$

$$j(h(x)) = j(2x-10) = 5(2x-10)^2$$

$$= 5(4x^2 - 40x + 100)$$

$$= 20x^2 - 200x + 500$$

(b) $(h \circ j)(x)$

$$h(j(x)) = h(5x^2) = 2(5x^2) - 10$$

$$= 10x^2 - 10$$