

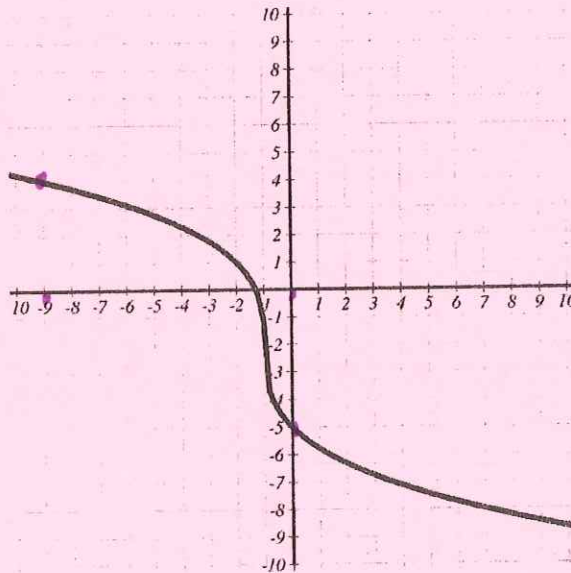
## Unit 2 Assessment C

Name: Answer Key

Learning Objective	Grade
Rates of Change	
Pointwise Behavior	
Global Behavior	
Graphing	
Rewriting Equations	
Function Algebra	
Inequalities	

## Functions as Quantities Changing with Each Other

30 pts

1. The function  $f(x)$  is graphed below:

- (a) What is the average rate of change of  $f$  over  $[-9, 0]$ ?

$$\frac{-5 - 4}{0 - (-9)} = \frac{-9}{9} = -1$$

- (b) The slope of the line tangent to the graph of  $f$  at  $x = -7$  is...
- Positive
- Negative
- Zero
- (c) The slope of the line tangent to the graph of  $f$  at  $x = -1.5$  is...
- Positive
- Negative
- Zero
- (d) If you compare the line tangent to the graph of  $f$  at  $x = -7$  and the line tangent to the graph of  $f$  at  $x = -1.5$ , which has a greater slope?
- The line tangent to  $f$  at  $x = -7$  has a greater slope.
- The line tangent to  $f$  at  $x = -1.5$  has a greater slope.
- The lines tangent to the graph of  $f$  at  $x = -7$  and  $x = -1.5$  have slopes that are equal.
- The graph doesn't give us enough information.
- (e) Describe how  $f$  is changing over  $(-\infty, -1)$ .
- increasing at a constant rate
- increasing at an increasing rate
- increasing at a decreasing rate
- decreasing at a constant rate
- decreasing at an increasing rate
- decreasing at a decreasing rate
- (f) Describe how  $f$  is changing over  $(-1, \infty)$ .
- increasing at a constant rate
- increasing at an increasing rate
- increasing at a decreasing rate
- decreasing at a constant rate
- decreasing at an increasing rate
- decreasing at a decreasing rate

2. Let  $f(x) = 6x^3$ . Find the average rate of change of  $f$  over  $[x, x + h]$ . You must show a step-by-step solution to receive credit.

30pts

$$ARC = \frac{6(x+h)^3 - 6x^3}{h}$$

$$= \frac{6(x^3 + 3x^2h + 3xh^2 + h^3) - 6x^3}{h}$$

$$= \frac{6x^3 + \overset{18}{3}x^2h + \overset{18}{3}xh^2 + \overset{6h^3}{h^3} - 6x^3}{h} = \frac{h(\overset{18}{3}x^2 + \overset{18}{3}hx + \overset{6h^2}{h^2})}{h}$$

~~$$= 3x^2 + 3hx + h^2$$~~

$$= 18x^2 + 18hx + 6h^2$$

10pts know diff quotient  
15pts use Khayyam's triangle  
5pts correct work

40 pts

3. Select **ONE** of the two following problems to work. You must show a step-by-step solution to receive credit. You must circle the letter of the one that you are working.

If you attempt both and expect your instructor to grade the better one, you will receive **NO CREDIT**.

If you attempt both, you **MUST** circle the letter of the one that you want your instructor to grade.

(a) Let  $f(x) = \sqrt{x}$ . Find the average rate of change of  $f$  over  $[x, x+h]$ .

(b) Let  $f(x) = \frac{1}{x}$ . Find the average rate of change of  $f$  over  $[x, x+h]$ .

$$(a) \text{ AR} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \boxed{\frac{1}{\sqrt{x+h} + \sqrt{x}}}$$

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$$(b) \text{ ARC} = \frac{\frac{x}{x} \cdot \frac{1}{x+h} - \frac{1}{x} \cdot \frac{x+h}{x+h}}{h} = \frac{\left(\frac{x-(x+h)}{x(x+h)}\right)}{h} = \frac{\left(\frac{-h}{x(x+h)}\right)}{h}$$

$$= \boxed{\frac{-1}{x(x+h)}}$$

## Behavior of Function at a Point

4. The function  $f$  is  $f(x) = 5(x-4)(x+8)^2(3x+1)$  when written in factored form and  $f(x) = 15x^4 + 185x^3 + 60x^2 - 3840x - 1280$  when  $f$  is written in standard descending order. Find each of the following:

30 pts 10 (a) The  $y$ -intercept

$$f(0) = -1280$$

$$(0, -1280)$$

20 (b) The zeros and their multiplicities

zero	4	-8	$-\frac{1}{3}$
mult	1	2	1

30 pts

5. The point(s) on the graph of  $f(x) = \frac{5x+1}{x+4}$  where  $y$  is  $\frac{7}{9}$ .

$$\frac{5x+1}{x+4} = \frac{7}{9}$$

$\Rightarrow$

$$9(5x+1) = 7(x+4)$$

$$45x+9 = 7x+28$$

$$38x = 19$$

$$x = \frac{19}{38}$$

$$x = \frac{1}{2}$$

$$\left(\frac{1}{2}, \frac{7}{9}\right)$$

40 pts

6. Let  $g(x) = \frac{(x+5)^2(x-1)}{5x(5x-3)(x-1)}$ . Find each of the following or state that none exist.

(a) The horizontal asymptote

End-Behavior!  $\frac{x^3}{25x^3} = \frac{1}{25}$

$$y = \frac{1}{25}$$

(b) The vertical asymptote(s)

$$x = \frac{3}{5}, \quad \text{[scribble]}, \quad x = 0$$

(c) The x-intercept(s)

$$(-5, 0), \quad \text{[scribble]}$$

(d) Any holes in the graph of  $g$ .

$$\left(1, \frac{18}{5}\right)$$

at  $x=1$ :  $\frac{(1+5)^2}{5 \cdot 1(5-3)} = \frac{36}{5 \cdot 2} = \frac{36}{10} = \frac{18}{5}$

Behavior of Function Over an Interval

40pts

7. Let  $g(n) = \frac{1}{n-2} + 9$ . Find each of the following. Hint: It may be helpful to envision the function's graph or create a quick sketch of it.

(a)  $\lim_{n \rightarrow -\infty} g(n) = 9$

(b)  $\lim_{n \rightarrow 2^-} g(n) = -\infty$

(c)  $\lim_{n \rightarrow 2^+} g(n) = \infty$

(d)  $\lim_{n \rightarrow 2} g(n) = \text{Does Not Exist}$

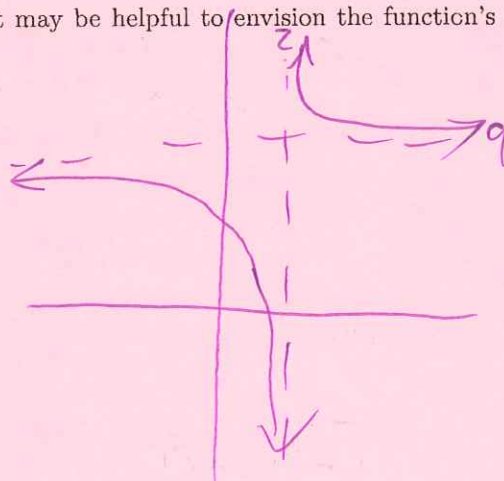
(e)  $\lim_{n \rightarrow \infty} g(n) = 9$

(f) Over the interval  $(-\infty, 2)$   $g$  is...

- concave up
- concave down
- neither

(g) Over the interval  $(2, \infty)$   $g$  is...

- concave up
- concave down
- neither



20pts

8. Let  $k(t) = \frac{t+6}{(t+6)(t-10)}$ . Find each of the following.

(a) The domain of  $k$

$t \neq -6 \text{ and } t \neq 10$ ;  $(-\infty, -6) \cup (-6, 10) \cup (10, \infty)$

(b) The hole in the graph of  $k$ .

at  $t = -6$ :  $\frac{1}{-16}$   $(-6, -\frac{1}{16})$

(c)  $k(-6) =$

Undefined

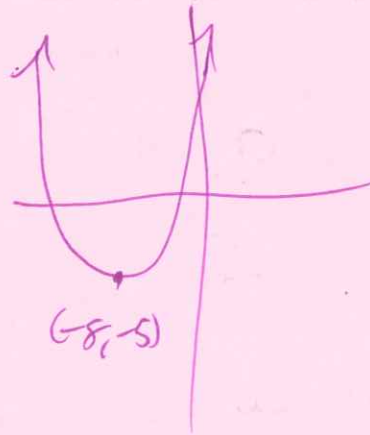
(d)  $\lim_{t \rightarrow -6} k(t) =$

$-\frac{1}{16}$

repts

9. Let  $f(x) = (x + 8)^4 - 5$ . State each of the following.(a) The domain of  $f$ .

$$(-\infty, \infty)$$

(b) The range of  $f$ .

$$[-5, \infty)$$

(c) Complete the following sentence:

The function  $f$  has an absolute minimum/maximum (circle one) of -5 at  $x =$  -8.

repts

10. Let  $p(x) = -\sqrt{x+4} - 9$ .(a) The domain of  $p$ .

Need  $x+4 \geq 0$   
 $x \geq -4$

$$[-4, \infty)$$

(b) As  $x \rightarrow \infty$ ,  $p(x) \rightarrow$   $-\infty$ (c)  $p(x)$  is

- increasing  
 decreasing  
 constant

(d)  $p(x)$  is

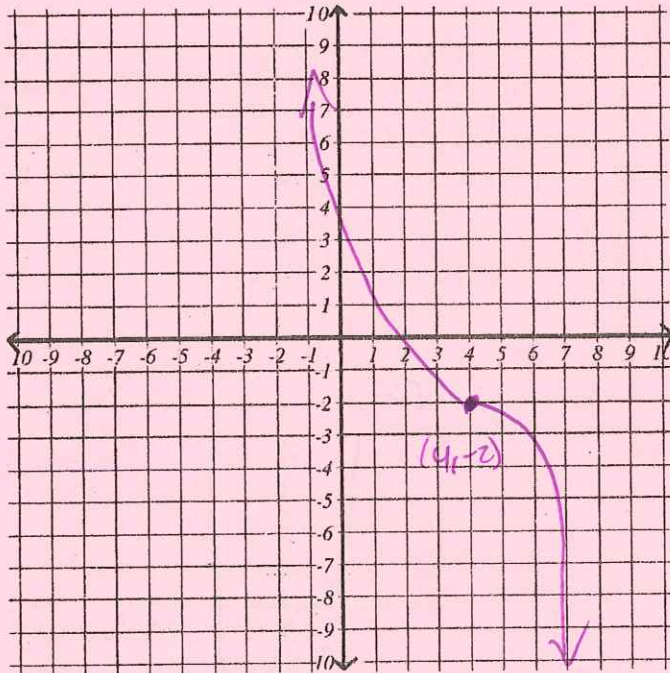
- concave up  
 concave down  
 neither



Graphs of Functions

20pts

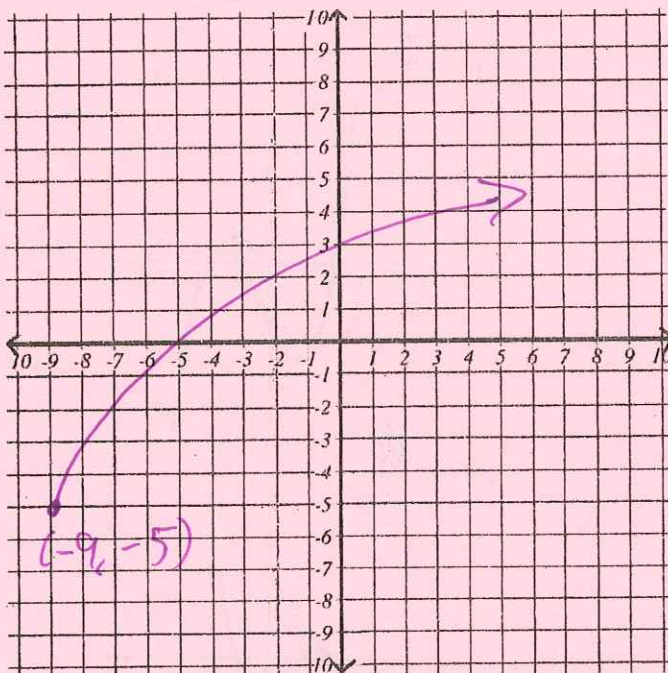
11. Graph the function  $f(x) = -(x - 4)^3 - 2$ .



for Each:  
10 slope/orientation  
40 shifts

20pts

12. Graph the function  $f(x) = -5 + \sqrt{x + 9}$ .



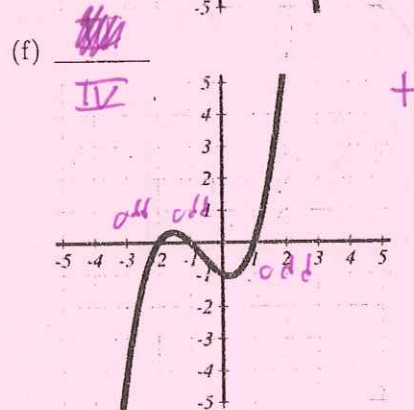
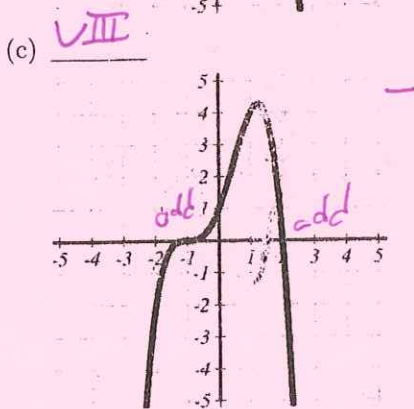
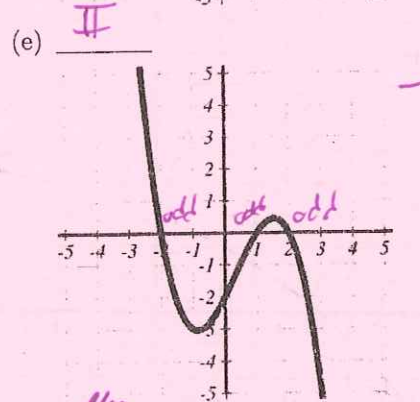
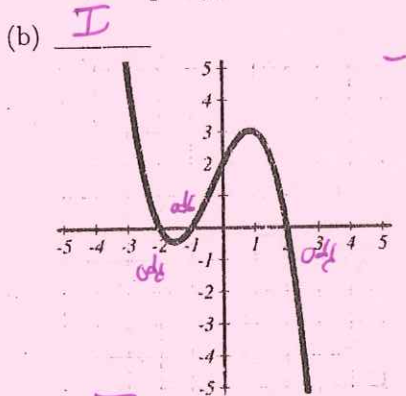
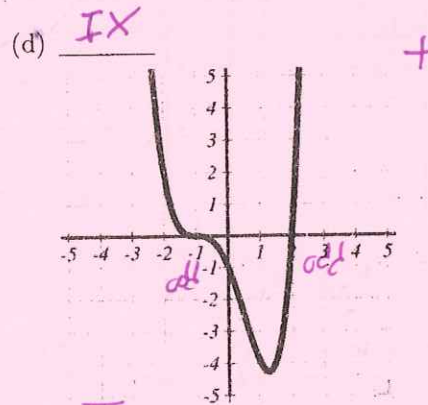
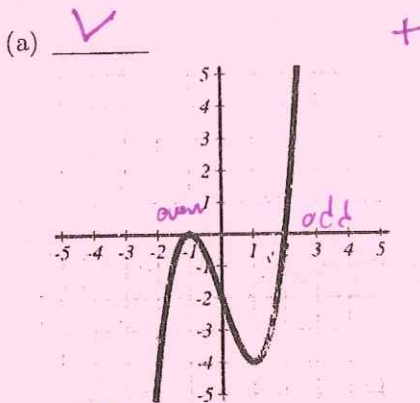
30pts

13. Match each of the following equations in the answer bank with the graphs in the questions below. Not all answers will be used.

**ANSWER BANK:**

- ~~(I)~~  $y = -(x+2)(x+1)(x-2)$    
 ~~(IV)~~  $y = (x+2)(x-1)(x+1)$    
 (VII)  $y = \frac{1}{2}(x-1)^3(x+2)$   
 (II)  $y = -(x+2)(x-1)(x-2)$    
 ~~(V)~~  $y = (x+1)^2(x-2)$    
 ~~(VIII)~~  $y = -\frac{1}{2}(x+1)^3(x-2)$   
~~(III)~~  $y = (x+2)(x+1)(x-2)$    
 (VI)  $y = (x-1)^2(x+2)$    
 ~~(IX)~~  $y = \frac{1}{2}(x+1)^3(x-2)$

**QUESTIONS:**



14. Match each of the following equations in the answer bank with the graphs in the questions below. Not all equations will be used, but no equation will be used more than once.

30 pts

ANSWER BANK:

(I)  $y = \frac{x+2}{x-2}$

(II)  $y = \frac{x-2}{x+2}$

(III)  $y = \frac{x-2}{(x+2)^2}$

(IV)  $y = \frac{(x+2)^2}{(x-2)^2}$

(V)  $y = \frac{x-2}{x(x+2)}$

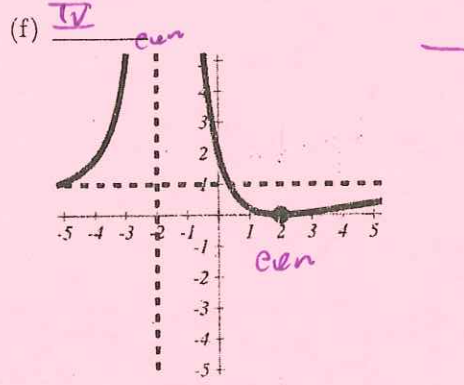
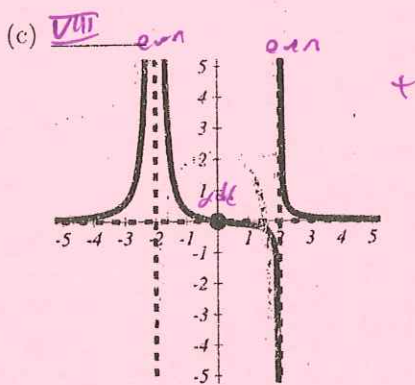
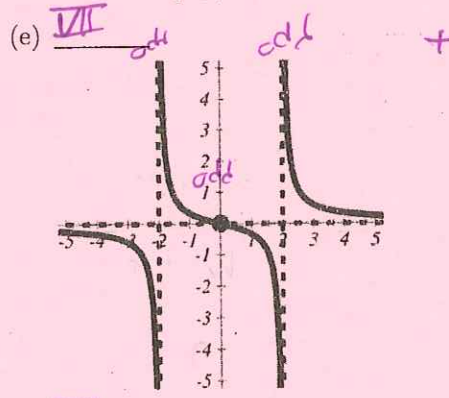
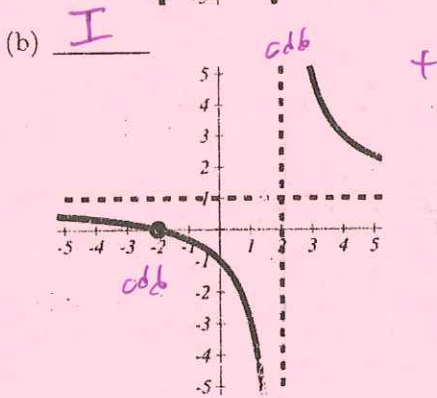
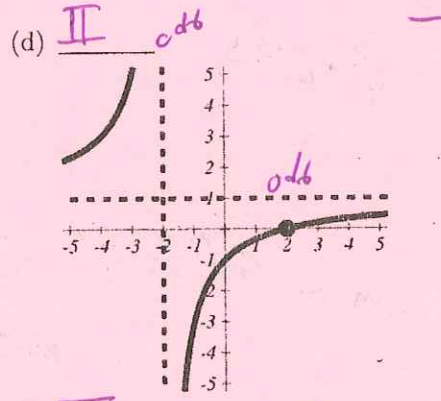
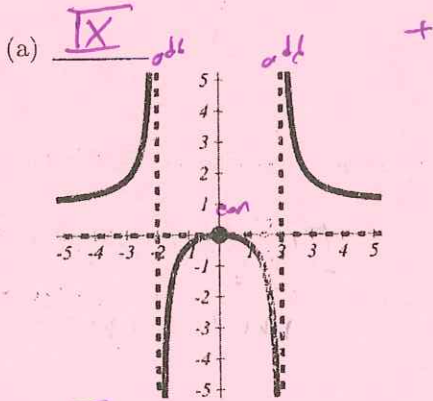
(VI)  $y = \frac{1}{(x-2)^2(x+2)}$

(VII)  $y = \frac{x}{(x-2)(x+2)}$

(VIII)  $y = \frac{x}{(x-2)(x+2)^2}$

(IX)  $y = \frac{x^2}{(x-2)(x+2)}$

QUESTIONS:



## Rewriting Equations of Functions

15. Rewrite each of the following in the form  $y = ax^n$ .

(a)  $f(x) = 9\sqrt{x}$

$$9x^{1/2}$$

(b)  $h(t) = \frac{1}{7t^4}$

$$\frac{1}{7} \cdot t^{-4}$$

(c)  $r(a) = \frac{7}{4\sqrt[5]{a^3}}$

$$\frac{7}{4} \cdot a^{-3/5}$$

16. Fully simplify  $k(m) = \frac{5m^2 - 20}{m^2 + 7m - 18}$ . Make sure that your final answer has the correct domain.

$$k(m) = \frac{\cancel{5m^2 - 20}}{\cancel{(m-2)}(m+9)} = \frac{5(m-2)(m+2)}{(m-2)(m+9)}$$

$$= \frac{5(m+2)}{m+9}, \quad (m \neq 2)$$

## Function Algebra

17. Let  $f(x) = 4 + \sqrt[3]{x+5}$ . Find the equation for  $f^{-1}$ .

60 pts

$$y = 4 + \sqrt[3]{x+5}$$

$$y - 4 = \sqrt[3]{x+5}$$

$$(y-4)^3 = x+5$$

$$(y-4)^3 - 5 = x$$

$$f^{-1}(y) = (y-4)^3 - 5$$

OR

$$f^{-1}(x) = (x-4)^3 - 5$$

40 pts

18. Assume  $f(x) = 6x^2$ ,  $g(x) = 8x^3$ , and  $k(x) = 2x^{-7}$ . Find each of the following. Write your final answers simplified with no negative exponents. Make sure that your final answer has the correct domain.

(a)  $(f \cdot g)(x)$

~~$f(x) = 6x^2$   
 $g(x) = 8x^3$   
 $(f \cdot g)(x) = 6 \cdot (8x^3)^2$   
 $= 6 \cdot 64 \cdot x^6$   
 $= 384x^6$~~

*this is wrong*

(a)

$f(x)g(x)$

$= 6x^2 \cdot 8x^3$

$= 48x^5$

(b)  $(\frac{f}{g})(x)$

$$\frac{f(x)}{g(x)} = \frac{6x^2}{8x^3} = \boxed{\frac{3}{4x}}$$

(c)  $(\frac{g}{k})(x)$

$$\frac{g(x)}{k(x)} = \frac{8x^3}{2x^{-7}} = \boxed{4x^{10} \quad (x \neq 0)}$$

(d)  $(f \circ k)(x)$

$$f(k(x)) = f(2x^{-7}) = 6(2x^{-7})^2 = 6 \cdot 4 \cdot x^{-14}$$

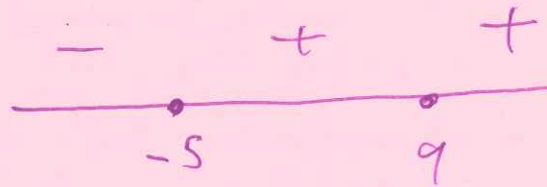
$$= 24x^{-14} = \boxed{\frac{24}{x^{14}}}$$

## Inequalities and Functions

40 pts

19. Create a sign diagram for the function  $f(x) = (x + 5)(x - 9)^2$ .

Z	M
-5	1
9	2



End Behavior:  
 $x^3$  ↓ ↗

60 pts

20. The sign diagram for  $f(x) = \frac{x-2}{x+7}$  is given below:

Find each of the following using the sign diagram:

(a) What is the domain of  $k(x) = \sqrt{\frac{x-2}{x+7}}$ ?Need  $f(x) \geq 0$ 

$$(-\infty, -7) \cup [2, \infty)$$

(b) Where is the function  $f(x) = \frac{x-2}{x+7}$  below the  $x$ -axis?

$$\text{Need } f(x) < 0; \quad (-7, 2)$$