

Math 1410: Study Guide for Midterm 1

September 12, 2022

General comments:

- The exam is 15 questions, ranging in length from asking you to look at a graph and identify a few features to asking you to do symbolic/numeric calculations.
- It's important to distinguish between the different types of objects we're dealing with—numbers, points, functions, and intervals—and understand what's being asked about. For example, if a problem asks you to find the y -intercept of a function, that is a *point*, while a problem asking you to find the initial value of a function was a *number*.
- Questions about domains, ranges, where a function is increasing, etc. are asking for intervals, and you should write your answers in interval notation. For example, if asked to find the range of the function $f(x) = x^2 - 6$ you should answer $[-6, \infty)$.
- The questions are written with the fact that calculators are not allowed in mind, so any numbers involved should be relatively small and manageable. If you need to use the quadratic formula, you're more likely to get an answer like $\frac{1 \pm \sqrt{3}}{2}$ than you are one like $\frac{-13 \pm \sqrt{105}}{8}$. (If you start to get something like the second one, you should take a moment to check your work to see whether you went wrong earlier!)
- Show your work! For one, understanding the process and how to communicate your logic to others is more important than being able to produce a correct answer with no explanation. For another, I cannot give partial credit if you show no work.

The problems on the exam are broken up by learning objective. As a general comment, the two types of functions we have looked at are quadratic functions and linear functions. So you should know how to work with both of these, for all learning objectives. Here's what you should know for each objective.

1. Function algebra.

- Given formulas for two functions, compute a formula for their sum, difference, product, quotient, or composition.
- Determine the behavior of a function at a point, where that function is given by sums, products, composition, etc. of other functions.

2. Pointwise behavior of functions.

- How to find the initial value and y -intercept of functions.
- How to find zeroes and x -intercepts of functions
- How to find an output/ y -value of a function given an input, and how to find the input(s)/ x -value(s) of a function given an output.
- For all of these, you should know how to work if the function is given as a formula, graph, or table. For functions given by a formula, you should know what to give both a number or a constant/expression.
- For quadratic functions, when and how to use completing the square or the quadratic formula to solve these problems.

3. Rewriting equations of functions.

- How to rewrite quadratic functions from $ax^2 + bx + c$ form to $a(x - h)^2 + k$ form, and vice versa.
- How to factor quadratic expressions.

4. Rates of change.

- Describe whether a function is increasing, decreasing, or constant on an interval or at a point. Determine whether the rate of change of a function is increasing, decreasing, or constant. For these, you should be able to work with both functions as graphs or functions as tables.
- Know the difference quotient formula for calculating average rate of change. Know how to apply it to find the average rate of change of a quadratic function over an interval. You should be able to work both with numbers as the endpoints and if they are given in terms of constants/expressions. In particular, expect to see a problem asking about average rate of change over an interval $[x, x+h]$.
- Understand the relationship between average rate of change and secant lines, and between instantaneous rate of change and tangent lines. Given graphs of these lines, be able to answer questions about them.

5. Graphing.

- Be able to sketch a graph of a quadratic function, identifying the vertex and intercepts.
- How to sketch a graph of a line.
- How to sketch a graph of a piecewise-defined function, where the pieces are linear functions.

6. Global behavior of functions.

- How to find the vertex of a quadratic function, where the function is given as a formula. Given the vertex of a quadratic function, how to determine its domain, range, where it is increasing/decreasing, and where it is concave up/concave down.
- Given a graph of a function, determine its domain, range, where it is increasing/decreasing, and where it is concave up/concave down.

7. Inequalities and functions.

- Understand how intervals express an inequality. For piecewise-defined functions, understand how the inequalities giving the cases tell you on what input domain to use each piece.

8. Transcendental operations.

- *This will not be part of this exam.*