MATH 321: IN-CLASS WORKSHEET 12
FRIDAY, APRIL 9TH

In class we talked about the Euler characteristic of planar graphs. Let me recall for you the definitions. A *planar graph* is a graph drawn on a sphere where the edges don't overlap. (Equivalently you can draw it on the plane.) This divides the square up into faces, the regions between edges. The Euler characteristic is $v - e + f$, where v is the number of vertices, e is the number of edges, and f is the number of faces. We proved that $v - e + f = 2$ for any planar graph.

For this worksheet, I want you to think about graphs drawn on a torus, a donut-shaped surface. One way to think of a torus is this: consider a square sheet of infinitely flexible paper. Bend the paper so that two opposite edges touch, making a cylinder. Then bend the two circles and the end of the cylinder so they touch. This can be hard to do anything hands on with, so here's a way to think of it with a flat plane. Draw a square, and imagine moving inside the square. Whenever you go off the left edge, you wrap around to the right edge, and vice versa. Whenever you go off the top edge, you wrap around to the bottom edge, and vice versa.

If a graph can be drawn on a torus without edges overlapping, call it a *torus graph*. Just like with a graph drawn on a sphere, it breaks the torus up into vertices, edges, and faces, and we can compute its Euler characteristic.

(1) Draw some torus graphs and compute their Euler characteristics. Do all torus graphs have the same Euler characteristic? Or can you get different Euler characteristics?
(2) Try to identify a pattern for which torus graphs have which Euler characteristic. Formulate this as a conjecture. Can you prove some or all of your conjecture?
(3) Submit on gradescope a short (1 to 2 paragraph) summary of what your group did for this worksheet.