MATH 1410: WORKSHEET FOR 4/28

Sign diagrams, graphs, and inequalities for rational functions

Making a sign diagram for a rational function is much like doing it with a polynomial, except there are extra complications to handle.

(1) In addition to zeroes, you also need to locate asymptotes and holes, and determine their multiplicities.
(2) There's more possibilities for what the end behavior looks like.

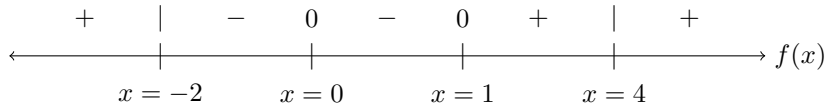
$$f(x) = \frac{3x^2(x-1)^5}{(x+2)^3(x-4)^2}$$

For the end behavior, you look at the leading terms: $\frac{3x^7}{x^5} = 3x^2$, so $f(x)$ goes to $+\infty$ in both directions.

To make the sign diagram, first notice this function has no holes, as there are no terms in common in the top and bottom. So you can make a table of zeros/asymptotes and their multiplicities:

	?	Mult
-2	A	3
0	Z	2
1	Z	5
4	A	2

This is then enough information to create the sign diagram:



And once you have the sign diagram you can sketch a graph or answer inequality questions. Remember: the sign diagram has all the information, so your main goal is to produce the sign diagram. Also, it's important to distinguish zeroes versus asymptotes versus holes in your sign diagram, since they behave differently when you're using the sign diagram to get information.

For example, suppose you want to solve $f(x) \leq 0$. The solution would include the two intervals where $f(x)$ is negative, plus the zeroes. (You don't include the asymptotes, because $f(x)$ is undefined there and so cannot be ≤ 0.) So the solution would be:

$$(-2, 1]$$

For each of the following rational functions, make a sign diagram. For a couple of these, you will need to first write them as a single fraction in factored form.

(1) $a(x) = \dfrac{-(x-2)^2(x+2)}{x^3}$.

(2) $b(x) = \dfrac{1}{x} + \dfrac{x}{1+2x}$.

(3) $c(x) = \dfrac{x^3 - 4x}{x(x+4)^2}$.

(4) $d(x) = \dfrac{(x-1)(x+2)^2(x-3)^3}{(x+1)^4(x-2)^3(x+3)^2}$.

(5) $f(x) = \dfrac{x^2(x-4)(x+4)^2}{x^3(x-4)(x+2)^4}$.

Solve the following inequalities. Give your answers in interval notation.

(1) $\dfrac{x^2-4}{x} > 0$.

(2) $\dfrac{x^2(x-2)(x+2)}{(x-1)^2(x+1)^3} \leq 0$.

Find the domains of the following functions. Give your answers in interval notation.

(1) $g(x) = \ln\left(\dfrac{x^2-1}{x^2+1}\right)$.

(2) $h(x) = \sqrt{\dfrac{x^4(x-2)}{(x+2)^2(x-4)}}$.