# Math 1410 Exam 1

Friday, Feb 24

Name: Answer Key

This is the first midterm exam.

Carefully read each question and understand what is being asked before you start to solve the problem. **Show your work in an orderly fashion, and circle or mark in some way your final answers.**

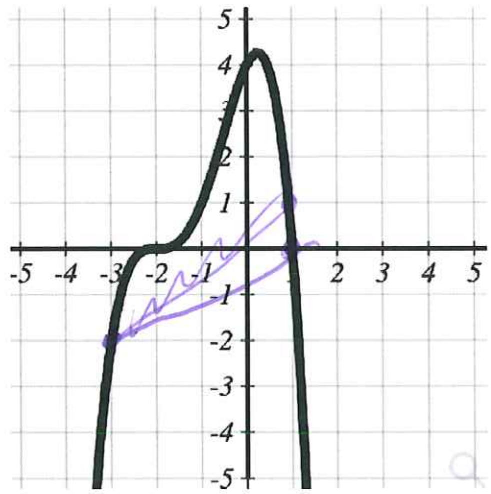
**No calculators nor other electronic devices are allowed.**

| **Learning Objective** | **Grade** |
|---|---|
| Functions as Covariation | |
| Pointwise Behavior | |
| Large Scale Behavior | |
| Graphs of Functions | |
| Function Algebra | |
| Evaluation and Rewriting | |

## Functions as Covariation

1. (40 points) The graph of the function $f(x)$ is shown below.

5 each




- What is the average rate of change of $f(x)$ over the interval $[-3, 1]$?

  $\frac{0-(-2)}{1-(-3)} = \frac{3}{4} = \boxed{\frac{1}{2}}$

- On the interval $(-\infty, -2)$ is $f(x)$ increasing or decreasing?

  increasing

- At $x = -2$, is the slope of the tangent line positive, negative, or zero?

  Zero

- On the interval $(-\infty, -2)$ is the rate of change of $f(x)$ increasing or decreasing?

  decreasing

- At $x = -1$, is the slope of the tangent line positive, negative, or zero?

  Positive

- On the interval $(1/2, \infty)$ is $f(x)$ increasing or decreasing?

  decreasing

- At $x = 0$, is the slope of the tangent line positive, negative, or zero?

  Positive

- On the interval $(1/2, \infty)$ is the rate of change of $f(x)$ increasing or decreasing?

  decreasing

2. (30 points) What is the average rate of change of $\ell(t) = 2\log_2(3t-2)$ across the interval $[1, 6]$? Give an exact answer and fully simplify.

$$ARC = \frac{\ell(6) - \ell(1)}{6-1} = \frac{2\log_2(16) - 2\log_2(\overset{1}{\cancel{\phantom{4}}})}{5}$$

$$= \frac{2 \cdot 4 - \overset{0}{\cancel{\phantom{2}}}}{5} = \boxed{\frac{\cancel{\phantom{4}}\,8}{5}}$$

3. (30 points) What is the average rate of change of $q(x) = x^2 - 3x + 1$ across the interval $[x, x+h]$? Simplify your answer fully.

$$ARC = \frac{(x+h)^2 - 3(x+h) + 1 - [x^2 - 3x + 1]}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 3x - 3h + 1 - x^2 + 3x - 1}{h}$$

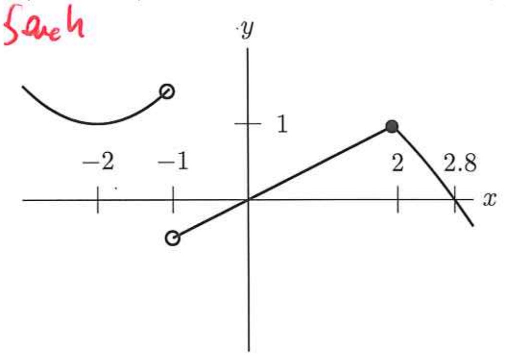
$$= \frac{2xh + h^2 - 3h}{h}$$

$$= \boxed{2x - 3 + h}$$

## Pointwise Behavior

4. (25 points) The graph of the function $f(x)$ is shown below.

5 each




Use this graph to determine the following.

(0,0) : The $y$-intercept.

Undefined : $f(-1)$

(0,0), (2.8,0) : The $x$-intercept(s).

$-2$, ~~2~~ 2 : All values of $x$ so that $f(x) = 1$.

1 : $f(-2)$

5. (25 points) Find all $x$- and $y$-intercepts of the function $g(x) = -2(x+1)^4 + 32$. [Hint: $2^4 = 16$.]

(0, 30) : The $y$-intercept. 10 pts

(−3,0), (1,0) : The $x$-intercept(s). 15 pts

$g(0) = -2 \cdot 1^4 + 32 = 30$

$-2(x+1)^4 + 32 = 0$

$32 = 2(x+1)^4$

$16 = (x+1)^4$

$\pm 2 = x+1$

$x = -1 \pm 2$

$= -3, 1$

6. (25 points) Find all $x$- and $y$-intercepts of the function $h(x) = e^2 - e^{-2x}$. Give exact answers.

$(0, e^2-1)$: The $y$-intercept. 10pts

$h(0) = e^2 - e^0 = e^2 - 1$

$(-1, 0)$: The $x$-intercept(s). 15pts

$0 = e^2 - e^{-2x}$

$e^{-2x} = e^2$

$-2x = 2$

$x = -1$

7. (25 points) Find all $x$- and $y$-intercepts of the function $j(x) = x^2 + 3x + 5$.

$(0, 5)$: The $y$-intercept. 10pts

$j(0) = 5$

No x-int.: The $x$-intercept(s). 15pts

$x^2 + 3x + 5 = 0$

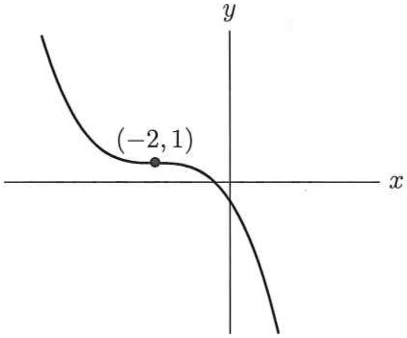
$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 5}}{2}$$

$$= \frac{-3 \pm \sqrt{-11}}{2}$$

No solution

## Large Scale Behavior

10 each 8. (50 points) The graph of $d(x) = -(x+2)^3 + 1$ is below.




Find all inflection points of $d(x)$, determine the interval on which it is increasing, the interval on which it is decreasing, the interval on which it is concave up, and the interval on which it is concave down.

$(-2,1)$: inflection point?

$\phi$: increasing?

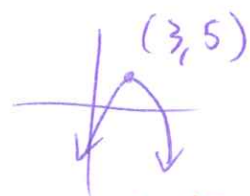
$(-\infty,\infty)$: decreasing?

$(-\infty,-2)$: concave up?

$(-2,\infty)$: concave down?

~~10~~ each 5 9. (50 points) Consider the quadratic function $f(x) = -2(x-3)^2 + 5$. Find all maximums/minimums of $f(x)$. Determine the interval on which it is increasing, the interval on which it is decreasing, the interval on which it is concave up, and the interval on which it is concave down.

$(3,5)$: max/min? [circle which one]




25 for correct Picture

$(-\infty,3)$: increasing?

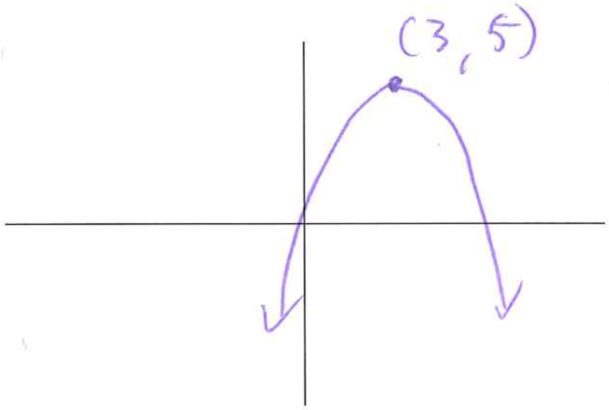
$(3,\infty)$: decreasing?

$\phi$: concave up?

$(-\infty,\infty)$: concave down?

## Graphs of Functions

10. (30 points) Consider the quadratic function $f(x) = -2(x-3)^2 + 5$. Sketch a graph of this function, clearly identifying where the vertex is.

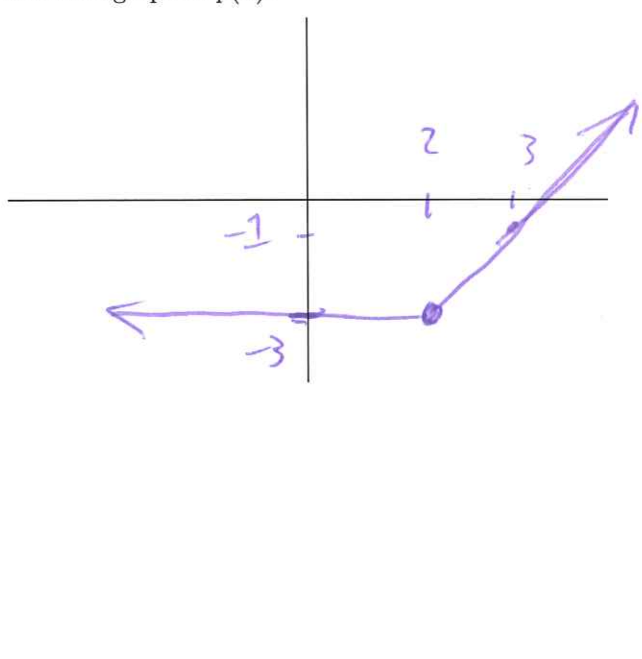



→3 ↑5
Vert. Flip

11. (30 points) Consider the piecewise linear function

$$p(x) = \begin{cases} -3 & x < 2 \\ 2x - 7 & x \geq 2. \end{cases}$$

Sketch a graph of $p(x)$.



12. (20 points) Match the following four functions with their graphs, out of the options given below.

5 each

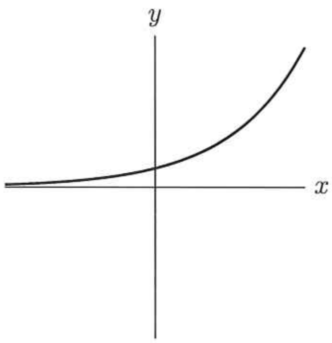
B: $e^{-x}$

C: $-e^{-x}$

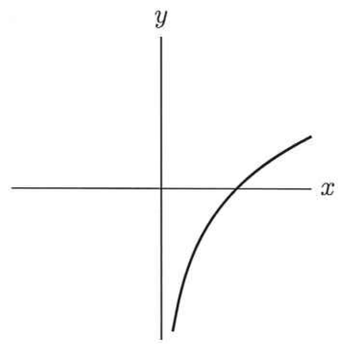
H: $\ln(-x)$

G: $-\ln(x)$

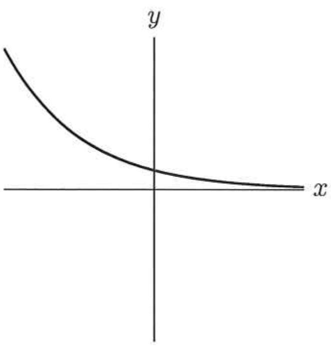
(A)



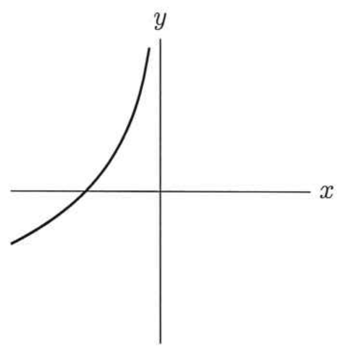

(E)



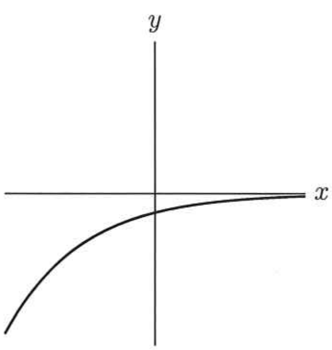

(B)



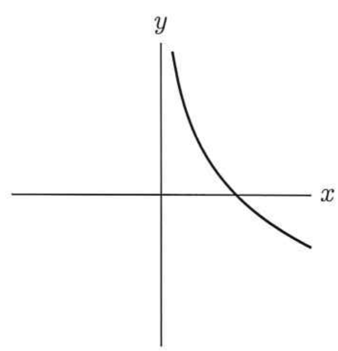

(F)



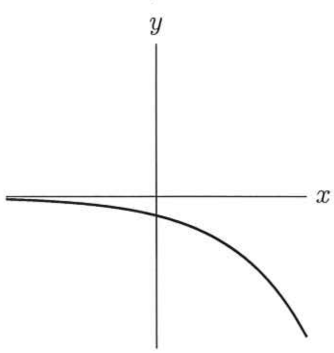

(C)



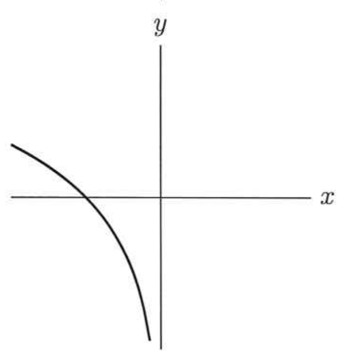

(G)




(D)




(H)



5 each

13. (20 points) Match the following four functions with their graphs, out of the options given below.

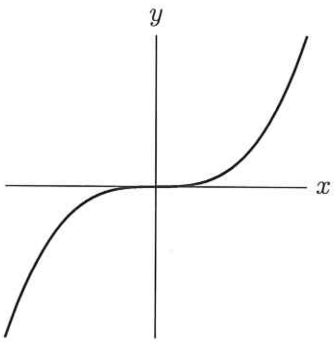
G: $\sqrt[3]{x}$

E: $\sqrt[4]{x}$

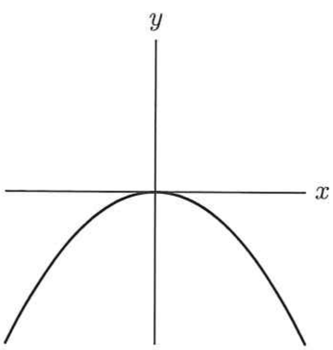
D: $x^4$

A: $x^3$

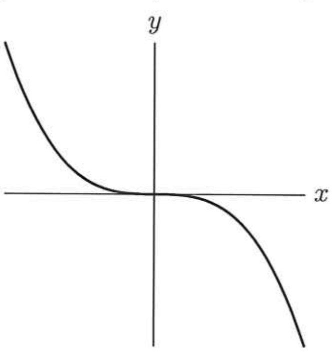
(A)



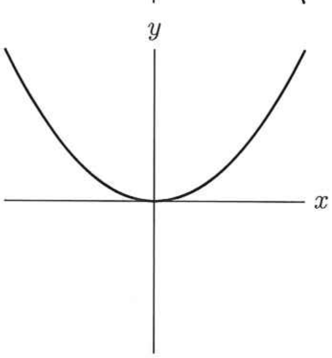

(B)



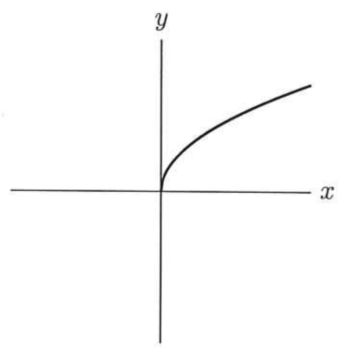

(C)



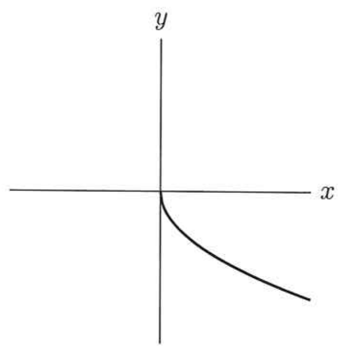

(D)



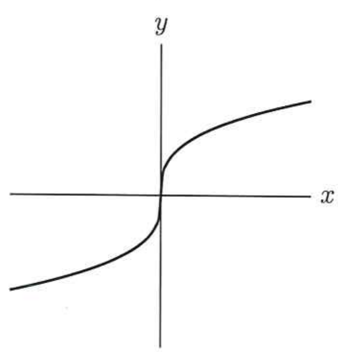

(E)



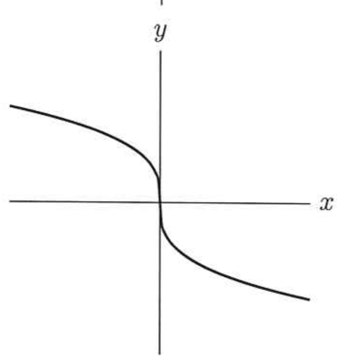

(F)




(G)




(H)



## Function Algebra

14. (50 points) Consider the function $a(x) = -(x+2)^3 - 8$. Find its inverse $a^{-1}(x)$.

$$y = -(x+2)^3 - 8$$

$$(x+2)^3 = -y-8$$

$$x+2 = \sqrt[3]{-y-8} = -\sqrt[3]{y+8}$$

$$x = -2 - \sqrt[3]{y+8}$$

$$\boxed{a^{-1}(x) = -2 - \sqrt[3]{x+8}.}$$

15. (50 points) Consider the function $b(x) = -\log_3(1-2t) + 1$. Find its inverse $b^{-1}(x)$.

$$y = -\log_3(1-2x)+1$$

$$\log_3(1-2x) = 1-y$$

$$1-2x = 3^{1-y}$$

$$-2x = 3^{1-y} - 1$$

$$x = \frac{1}{2} - \frac{1}{2} \cdot 3^{1-y}$$

$$\boxed{b^{-1}(x) = \frac{1}{2} - \frac{1}{2} \cdot 3^{1-x}}$$

## Evaluation and Rewriting

16. (30 points) Rewrite the quadratic function $f(x) = x^2 + 4x - 12$ in vertex form.

$f(x) = (x^2+4x+4)-4-12$

$\boxed{f(x) = (x+2)^2-16}$

17. (20 points) Rewrite the quadratic function $f(x) = x^2 + 4x - 12$ in factored form.

$\boxed{f(x) = (x+6)(x-2)}$

18. (30 points) Solve the following equation, giving an exact answer:

$$3^{2x+4} = 9^{3x-4}.$$

$\log_3(3^{2x+4}) = \log_3(9^{3x-4})$

$2x+4 = 3x-4 \cdot \underbrace{\log_3(9)}_{=2}$

$= 6x-8$

$12 = 4x$

$\boxed{3 = x}$

19. (20 points) Simplify the following expression to only contain a single logarithm:

$$3\log_7(x+1) + \log_7(x-1) - \log_7(x^2+1).$$

$$\boxed{= \log_7\left(\frac{(x+1)^3(x-1)}{x^2+1}\right)}$$