Math 1410 Exam 2

Friday, Apr 14

Name: Ansner Key

This is the second midterm exam.

Carefully read each question and understand what is being asked before you start to solve the problem. **Show your work in an orderly fashion, and circle or mark in some way your final answers.**

No calculators nor other electronic devices are allowed.

Learning Objective	**Grade**
Functions as Covariation	
Pointwise Behavior	
Large Scale Behavior	
Graphs of Functions	
Function Algebra	
Evaluation and Rewriting	

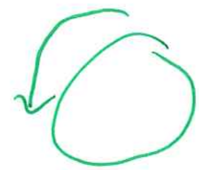
Functions as Covariation

1. (20 points) The angle θ sweeps across quadrant 1 in the positive direction. Is $\sin\theta$ increasing or decreasing? Is the rate of change increasing or decreasing? Please circle below.

   - $\sin\theta$ is **increasing** or **decreasing**.
   - The rate of change is **increasing** or **decreasing**.

2. (20 points) The angle θ sweeps across quadrant 2 in the positive direction. Is $\cos\theta$ increasing or decreasing? Is the rate of change increasing or decreasing? Please circle below.

   - $\cos\theta$ is **increasing** or **decreasing**.
   - The rate of change is **increasing** or **decreasing**.



3. (30 points) Determine the average rate of change of $s(x) = 3\sin(x/2)$ across the interval $[0, \pi]$. Give an exact answer.

$$ARC = \frac{3\sin(\frac{\pi}{2}) - 3\sin(0)}{\pi - 0} = \frac{3-0}{\pi} = \underline{\frac{3}{\pi}}$$

4. (30 points) Determine the average rate of change of $t(x) = x^4 + 2x$ across the generic interval $[x, x+h]$. Fully simplify your answer. [Hint: you can simplify it to no longer be a fraction.]

$$ARC = \frac{(x+h)^4 + 2(x+h) - (x^4 + 2x)}{h}$$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$$= \frac{\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 + \cancel{2x} + 2h - \cancel{x^4} - \cancel{2x}}{h}$$

$$= 4x^3\cancel{h} + 6x^2h + 4xh^2 + h^3 + 2$$

$$= \underline{4x^3 + 2 + 6x^2 + 4xh^2 + h^3}$$

Pointwise Behavior

5. (20 points) Find all zeroes of the polynomial

$$p(x) = -4x^2(x-2)(x+3)^3(x-5)^2.$$

$x = 0, 2, -3, +5$

6. (30 points) Find all x- and y- intercepts of the polynomial

$$q(x) = x^3 + 2x^2 - 8x.$$

y-ints (0,0)

$q(0) = 0$

x-ints: $\boxed{(-4,0) \quad (0,0) \quad (2,0)}$

$0 = x^3 + 2x^2 - 8x$

$= x(x^2 + 2x - 8)$

$0 = x(x+4)(x-2)$

$x = -4, 0, 2$

7. (20 points) Find all solutions to the following equation in the interval $[0, 2\pi]$. Please give exact answers.

slope=-1

$$3\tan x + 3 = 0$$

$\tan x = -1$

$x = \arctan(-1) + \pi K, \quad K \in \mathbb{Z}$

$x = -\frac{\pi}{4} + \pi K, \quad K \in \mathbb{Z}$

$\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}}$

$x = \ldots, -\frac{\pi}{4}, \underline{\frac{3\pi}{4}, \frac{7\pi}{4}}, \frac{11\pi}{4}, \ldots$

8. (30 points) Find the general solution to the following equation. Please give exact answers. [Hint: Your answer should depend upon a parameter K which can be any integer.]

$x=\frac{1}{2}$

$$6\cos(4x) = 3.$$

$\cos(4x) = \frac{1}{2}$

$4x = \pm \arccos(\frac{1}{2}) + 2\pi K, \quad K \in \mathbb{Z}$

$= \pm \frac{\pi}{3} + 2\pi K$

$\boxed{x = \pm \frac{\pi}{12} + \frac{\pi}{2} K, \quad K \in \mathbb{Z}}$

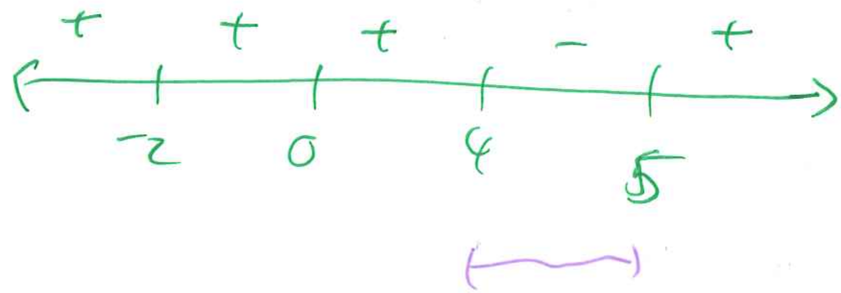
LS Behavior

#9 Solve the polynomial inequality. State in interval notation

$$3x^2(x-4)(x+2)^2(x-5)^3 < 0$$

End Behavior: $3x^8 \quad \nwarrow f \nearrow$

Z	M
-2	2
0	2
4	1
5	3



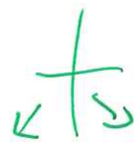
solution: $\underline{(4, 5)}$

10. (50 points) What is the domain of the following function? Give your answer in interval notation.

$$f(x) = \sqrt{-2x^4 + 8x^2}$$

domain where $-2x^4 + 8x^2 \geq 0$

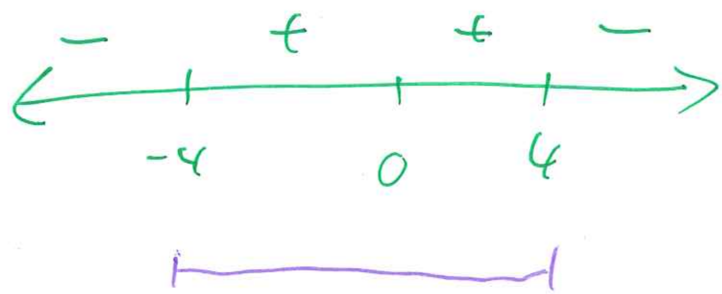
$-2x^4$



$-2x^2(x^2 - 4) \geq 0$

$-2x^2(x-4)(x+4) \geq 0$

Z	M
-4	1
0	2
4	1



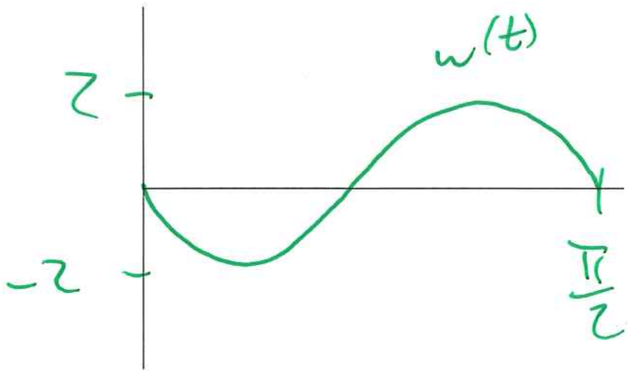
domain = [-4, 4]

Graphs of Functions

11. (30 points) A wave is modeled by the function

$$w(t) = -2\sin(4t).$$

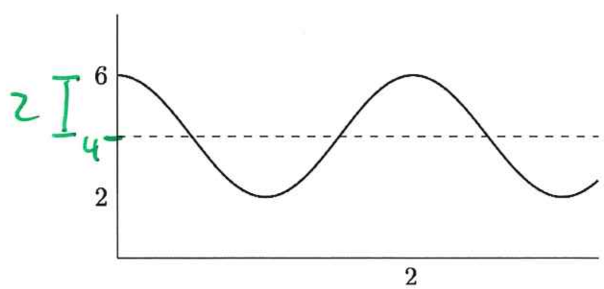
What are the amplitude and period of the wave? Use this info to sketch a graph of one period of the wave. Mark the amplitude and period on the axes.



$\text{Ampl} = 2$

$\text{Period} = \frac{2\pi}{4} = \frac{\pi}{2}$

12. (30 points) A wave oscillates between a minimum of 2 and a maximum of 6 with a period of 2. At time $t = 0$ the wave is at its maximum, as in the following graph. Use this information to write a function which models the wave. [Hint: Your answer can be in the form $A\texttt{trig}(Bt) + C$, where **trig** is either sin or cos.]



$C = \frac{6+2}{2} = 4$

$|A| = 6 - C = 2$

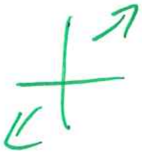
$\text{Period} = \frac{2\pi}{B} = 2$

$B = \frac{2\pi}{2} = \pi$

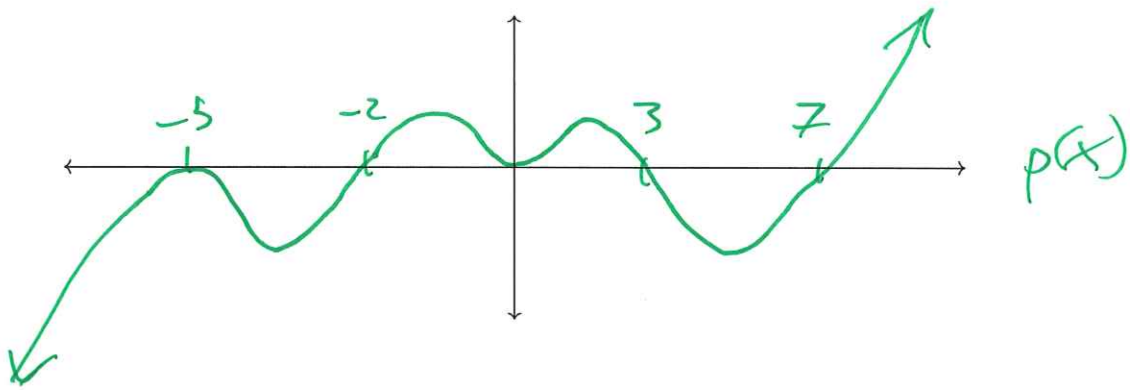
start at top, so +cos

$w(t) = 2\cos(\pi t) + 4$

13. (40 points) A polynomial $p(x)$ has leading term $4x^{11}$ and has the following sign diagram. Use this information to sketch a graph of $p(x)$. Your graph should make clear the end behavior of $p(x)$, where the zeroes of $p(x)$ are, and where $p(x)$ is positive versus negative.



$-$	0	$-$	0	$+$	0	$+$	0	$-$	0	$+$	$p(x)$
	$x=-5$		$x=-2$		$x=0$		$x=3$		$x=7$		



Function Algebra

14. (20 points) What is sin(arcsin(1/3))? Give an exact answer.

$\frac{1}{3}$

15. (20 points) Give an angle θ so that $\arccos(\cos\theta) \neq \theta$. Write a sentence or two explaining your answer.

$\theta = -\frac{\pi}{7}$.

arccos outputs an angle in $[0, \pi]$, so any θ outside this range will do.

16. (60 points) Consider the polynomials $a(x) = 2x^2 + x$ and $b(x) = x^3 - 2x^2$.

(a) Check that $a(x) + b(x)$ is a polynomial by computing the sum and simplifying.

$a(x)+b(x) = 2x^2 + x + x^3 - 2x^2$

$= \underline{x^3 + x}$

(b) Check that $a(x) \cdot b(x)$ is a polynomial by computing the product and simplifying.

$a(x) \cdot b(x) = (2x^2+x)(x^3-2x^2) = 2x^5 - 4x^4 + x^4 - 2x^3$

$= \underline{2x^5 - 3x^4 - 2x^3}$

(c) Check that $a(b(x))$ is a polynomial by computing the composition and simplifying.

$a(b(x)) = a(x^3-2x^2) = 2(x^3-2x^2)^2 + (x^3-2x^2)$

$= 2(x^6 - 4x^5 + 4x^4) + x^3 - 2x^2$

$= \underline{2x^6 - 4x^5 + 4x^4 + x^3 - 2x^2}$

Evaluation and Rewriting

17. (40 points) Give exact values for:

- $\sin(\pi) =$ -1
- $\csc(7\pi/6) =$ $\frac{1}{\sin(7\pi/6)} = \frac{1}{-1/2} = -2$

  Q3
- $\tan(\pi/4) =$ 1
- $\tan(0) =$ 0
- $\cos(-\pi/3) =$ $+\frac{1}{2}$

  Q4
- $\arccos(1) =$ π
- $\sec(2\pi/3) =$ $\frac{1}{\cos(\frac{2\pi}{3})} = \frac{1}{-1/2}$

  Q2

  $= -2$
- $\arcsin(-1/2) =$ $-\frac{\pi}{6}$

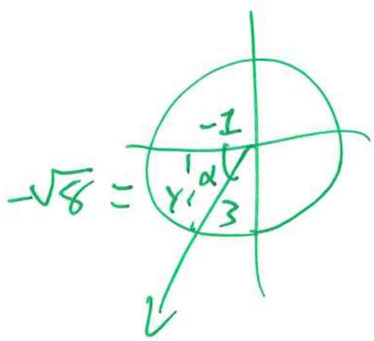
18. (15 points) The angle θ points to the point $(-0.6, -0.8)$ on the unit circle. Determine $\sin\theta$, $\cos\theta$, and $\tan\theta$.

$\sin\theta = \underline{-0.8}$

$\cos\theta = \underline{-0.6}$

$\tan\theta = \frac{-0.8}{-0.6} = \underline{\frac{4}{3}}$

19. (20 points) The angle θ is in quadrant 4 and has reference angle α. If $\cos(\alpha) = 1/3$, determine $\sin(\theta)$, $\sec(\theta)$, and $\tan(\theta)$.



$y^2 + 1^2 = 3^2$

$y = \sqrt{8}$

$\sin\theta = \underline{\dfrac{-\sqrt{8}}{3}}$

$\sec\theta = \underline{-\dfrac{3}{1}}$

$\tan\theta = \dfrac{-\sqrt{8}}{-1} = \underline{\sqrt{8}}$

20. (25 points) Fully simplify the trig expression

$$\frac{\sin x}{\csc x} + \frac{\cos^2 x}{\sin x \csc x} - \tan x \cot x.$$

$= \dfrac{\sin x}{1/\sin x} + \dfrac{\cos^2 x}{\sin x/\sin x} - \dfrac{\sin x}{\cos x} \cdot \dfrac{\cos x}{\sin}$

$= \sin^2 x + \cos^2 x - 1$

$= 1 - 1$

$= \underline{0}$