Math 1410: Study Guide for Midterm 1

General comments:

- The exam is about 20 questions, broken up by the six learning objectives. Each learning objective is scored out of 100, and I will report your grades separately by learning objective.
- Calculators and notes are not allowed for the exam. The questions are written with the fact that these are not allowed in mind. In particular, you should expect that any numbers involved in calculations will be relatively small and manageable.
- Show your work! For one, understanding the process and how to communicate your logic to others is more important than being able to produce a correct answer with no explanation. For another, I cannot give partial credit if you show no work.

So far we've studied these kinds of functions:

- Quadratic functions;
- Power and radical functions;
- Exponential functions and logarithms.

For the topics below you should understand how to apply them for each of these classes of functions.

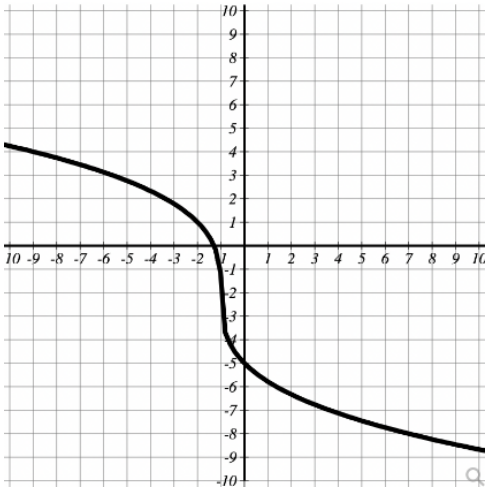
Here's what you should know for each learning objective.

1. Functions as Covariation
   - How the graph of a function relates to its rate of change; given a graph how to determine whether its rate of change is positive, negative, or zero; what it meanss for a function to be increasing at an increasing rate versus increasing at a decreasing rate, and similar for decreasing.
   - How to compute average rates of change, both over an interval with fixed endpoints and over the generic interval $[x, x + h]$.
2. Pointwise Behavior of Functions
   - How to find x- and y-intercepts of a function.
   - How to solve forward and inverse problems.
3. Large Scale Behavior of Functions
   - How to determine the domain and range of a function.
   - How to determine where a function is increasing or decreasing.
   - How to determine the concavity of a function.
   - How to find maximums, minimums, and inflection points.
4. Graphs of Functions
   - How to sketch a graph of a function.
   - How to identify a function's behavior from its graph.
   - How to graph a piecewise linear function.

5. Function Algebra
    - How to compute the inverse of a one-to-one function.
6. Evaluating and Rewriting Functions
    - How to complete the square to rewrite a quadratic function into vertex-form.
    - How to factor a quadratic function.
    - How to evaluate logarithms and exponential functions, and how to simply expressions involving them.

Here are some sample questions similar to what you should expect to see on the exam.

1. Consider the following graph of the function $F(x)$.



    - What is the average rate of change of $F(x)$ over the interval $[-9, 0]$?
    - Is the slope of the tangent line to the graph at $x = -3$ positive, negative, or zero?
    - What about at $x = -5$?
    - If you compare these two tangent lines, which one's slope is greater?
    - On the interval $(-\infty, -1)$ the function $F(x)$ is which: increasing or decreasing? At an increasing or decreasing rate?
    - What about on the interval $(-1, \infty)$.
2. Calculate the average rate of change of $f(x) = \sqrt[3]{x+4} - 1$ along the interval $[4, 23]$.
3. Calculate the average rate of change of $g(x) = 2x^2 + 3x$ along the interval $[x, x+h]$.
4. Calculate the average rate of change of $h(x) = 3^{2x-1}$ along the interval $[0, 2]$.
5. Find all x- and y-intercepts of $a(t) = t^2 - 4t - 12$.
6. Find all x- and y-intercepts of $b(t) = 3 - e^{1-t}$.
7. Find all x- and y-intercepts of $c(t) = 2(t+1)^3 - 1$.
8. Find all x- and y-intercepts of $d(t) = \sqrt{2-t} - 4$.
9. If $f(x) = x^2 + 4x - 3$ find all inputs x with an output of $f(x) = 2$.
10. Determine the domain and range of $f(x) = -2(x+2)^2 + 1$. Where is the function increasing? Where is it decreasing? Is it concave up or concave down?

11. What are the domain and range of $g(x) = 4 - 2 \cdot e^{2-x}$? Is the function increasing or decreasing? Concave up or concave down?

12. What is the domain of $h(x) = \sqrt[4]{3x-2} + 3$?

13. What are the domain and range of $j(x) = -\log_{1000}(5-x)$?

14. Find the vertex and any zeroes of $q(s) = -s^2 + 2s - 4$, and sketch a graph of q.

15. Identify what geometric transformations were applied to a basic power function x^n to get $f(x) = -3(x+1)^5 - 3$. Find the intercepts of $f(x)$ and sketch a graph, identifying the intercepts and inflection point.

16. Sketch a graph of $g(x) = 3 \cdot 2^{x-1}$, identifying all asymptotes. What are the domain and range of $g(x)$?

17. Sketch a graph of $h(x) = \log_3((2x-3)^2)$, identifying all asymptotes. What are the domain and range of $h(x)$?

18. Sketch a graph of the following piecewise-defined function:

$$\ell(x) = \begin{cases} 2x - 1 & x < 1 \\ 1 & x \geq 1 \end{cases}$$

19. Find the inverse of $b(x) = 3(x-1)^3 + 8$.

20. Find the inverse of $c(x) = e^{2x-2} + 3$.

21. Find the inverse of $d(x) = -\log_2(3x) - 3$.

22. Find the inverse of $f(x) = \sqrt{3x+1} - 1$. What is the domain of $f^{-1}(x)$?

23. Rewrite the quadratic expression $2x^2 - 10x + 6$ in factored form.

24. Rewrite the quadratic expression $2x^2 - 10x + 6$ in vertex form.

25. Fully simplify the following expression. All exponents should be positive.

$$(3x)^2 \cdot (xy)^{-7}$$

26. Use rules for logarithms to rewrite this expression to have a single log:

$$\log_2(x-1) + \log_2(x+1) - 2\log_2(x).$$

27. Fully simplify the following expression. Your final answer should not contain any logs.

$$\log_3\left(\frac{9^2}{3}\right)$$

28. Fully simplify the following expression. Your final answer should contain a single exponential.

$$\left(e^{2x}\right)^3 \cdot e^3$$

29. Solve the equation

$$2^{1-x} = 3^{2x}.$$