Math 1410: Study Guide for Midterm 2

General comments:

- The exam is about 20 questions, broken up by the six learning objectives. Each learning objective is scored out of 100, and I will report your grades separately by learning objective.
- Calculators and notes are not allowed for the exam. The questions are written with the fact that these are not allowed in mind. In particular, you should expect that any numbers involved in calculations will be relatively small and manageable.
- Show your work! For one, understanding the process and how to communicate your logic to others is more important than being able to produce a correct answer with no explanation. For another, I cannot give partial credit if you show no work.

For this unit we studied two kinds of functions:

- Trig functions; and
- Polynomials.

For the topics below you should understand how to apply them for each of these classes of functions.
Here's what you should know for each learning objective.

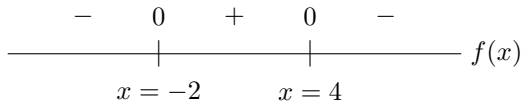
1. Functions as Covariation
   - How to compute average rates of change with trig functions and polynomials.
   - How to tell in which quadrants the rate of change of a trig function is positive or negative, and whether the rate of change is increasing or decreasing.
2. Pointwise Behavior of Functions
   - How to solve trig equations.
   - How to find zeroes of polynomials.
3. Large Scale Behavior of Functions
   - How to solve polynomial inequalities and determine where a polynomial is positive or negative.
   - How to determine the domains of functions involving polynomials.
4. Graphs of Functions
   - How to sketch a graph of a wave given a function.
   - Given a wave, how to determine a function that gives the wave.
   - How to sketch a graph of a polynomial given its sign diagram.
5. Function Algebra
   - Understand the relationship between arc trig functions and ordinary trig functions.
   - Understand how to check that the sum, difference, product, or composition of two polynomials is a polynomial. Understand how these operations determine the degree of the output polynomial.

6. Evaluating and Rewriting Functions

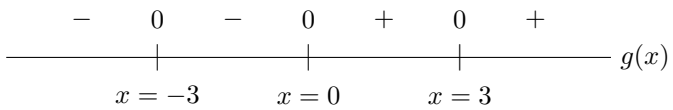
   - How to determine the values of trig function at special angles.
   - How to determine the values of trig functions knowing about the reference angle.
   - How to use the unit circle and XYR definitions of the trig functions to compute values.
   - How to use trig identities to simplify trig expressions.

Here are some sample questions similar to what you should expect to see on the exam.

1. Calculate the average rate of change of $a(x) = 3\cos(2x)$ on the interval $[0, \pi/3]$.
2. Calculate the average rate of change of $b(x) = x^3 - 2x^2 + 1$ on the generic interval $[x, x + h]$.
3. The angle θ sweeps across quadrant 3 in the positive direction. Is the rate of change of $\cos\theta$ positive or negative? Is the rate of change increasing or decreasing?
4. The angle θ sweeps across quadrant 4 in the positive direction. Is the rate of change of $\sin\theta$ positive or negative? Is the rate of change increasing or decreasing?
5. Find the general solution to $2\cos x + \sqrt{2} = 0$.
6. Find the general solution to $3\sin x + 2 = 4$.
7. Find the general solution to $\tan(3x) + 1 = 0$.
8. Find all x-intercepts of $p(x) = -2x^3(x + 4)(x - 2)^2(x - 10)$.
9. Find all x- and y-intercepts of $q(x) = 4x^5 + 2x^4 - x^3$.
10. Determine the domain of $f(x) = \ln(x^5 - 4x^4 + 4x^3)$. Give your answer in interval notation.
11. Solve the inequality $-2(x - 4)(x + 3)^2(x - 2)^3 \geq 0$. Give your answer in interval notation.
12. Determine the domain of $g(x) = 4x^3 - 3x^2 + 2x + 10$.
13. Sketch a graph of one full period of $w(x) = -2\sin(x/2)$. What are the amplitude and period of the wave?
14. A wave oscillates between a minimum of 0 and a maximum of 6 with a period of 2. If the wave starts in the middle moving upward at time 0, write a trig function which models the wave.
15. Suppose you know that a polynomial $f(x)$ has leading term $-2x^6$ and has the following sign diagram. Use this information to sketch a graph of $f(x)$.



16. Suppose you know that a polynomial $g(x)$ has leading term x^7 and has the following sign diagram. Use this information to sketch a graph of $g(x)$.



17. Give an angle θ so that $\sin(\theta) = 1/2$ but $\arcsin(1/2) \neq \theta$.

18. Let $p(x) = x^2 + 2x$ and $q(x) = x^3 - 1$. Check that $(p+q)(x)$, $(pq)(x)$, and $(p \circ q)(x)$ are all polynomials by writing them in the form

$$a_n x^n + \cdots a_1 x + a_0.$$

19. Suppose you know $a(x)$ is a polynomial with nonzero degree n and $b(x)$ is a polynomial with nonzero degree $m < n$. What is the degree of $(a + b)(x)$? What is the degree of $(ab)(x)$? What is the degree of $(a \circ b)(x)$?

20. Give the exact values for:

$$\begin{array}{ccc} \sin(0) & \cos(\pi) & \cot(\pi/4) \\ \csc(-\pi/3) & \sec(7\pi/6) & \tan(3\pi/2) \\ \arcsin(1) & \arccos(-\sqrt{2}/2) & \arctan(0) \end{array}$$

21. The angle θ is in quadrant 2 and has reference angle α. If $\cos(\alpha) = 3/5$, determine $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.

22. The angle θ points to the point $(5/13, -12/13)$ on the unit circle. Determine $\sec\theta$, $\tan\theta$, and $\sin\theta$.

23. Fully simplify the trig expression

$$\frac{\sin x}{\csc x} + \cot x \sin x.$$