Math 1410: Study Guide for Final Exam

General comments:

- The final exam is cumlative. About 1/4 of the material is new since the last midterm (so rational functions), and the remaining 3/4 is from the two midterms.
- Calculators and notes are not allowed for the exam. The questions are written with the fact that these are not allowed in mind. In particular, you should expect that any numbers involved in calculations will be relatively small and manageable.
- Show your work! For one, understanding the process and how to communicate your logic to others is more important than being able to produce a correct answer with no explanation. For another, I cannot give partial credit if you show no work.

This class we studied these classes of functions

- Quadratic functions;
- Power and radical functions;
- Exponential functions and logarithms;
- Trig functions;
- Polynomials; and
- Rational functions.

For the topics below you should understand how to apply them for each of these classes of functions.
Here's what you should know for each learning objective.

1. Functions as Covariation
   - How to compute average rates of change, both with set endpoints to the interval and on a generic interval $[x, x+h]$.
   - How to determine information about rates of change from a graph—where is the function increasing/decreasing, the difference between increasing at an increasing rate versus increasing at a decreasing rate, etc.
2. Pointwise Behavior of Functions
   - How to solve forward problems—given an input, find the output.
   - How to solve inverse problems—given an output, find the input(s).
3. Large Scale Behavior of Functions
   - How to determine domain of a function.
   - How to determine the range of a function (with the exception of polynomials and rational functions).
   - How to determine information about increasing/decreasing and concavity (again, with the exception of polynomials and rational functions).

- How to use sign diagrams to tell where a polynomial or rational function is positive/negative, and how to use this to solve inequalities.

4. Graphs of Functions

   - How to sketch a graph of a quadratic function, power function, radical function, exponential function, logarithm, sine/cosine function, or basic rational function (based on $1/x^n$).
   - Given the graph of a function, how to get information about large scale behavior of the function from it.
   - Given a graph of a polynomial or rational function, how to determine information about zeroes/asymptotes and their multiplicities.
   - How to sketch a graph of a pointwise-defined function (linear pieces only).

5. Function Algebra

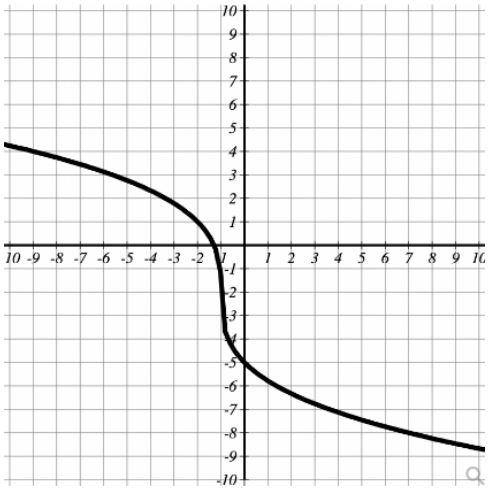
   - Understand how to use algebraic operations to combine functions to build up more complicated functions.
   - How to find the inverse of a function.
   - How the arc trig functions relate to the forward trig functions.

6. Evaluating and Rewriting Functions

   - The values of trig functions at special angles.
   - How to calculate values of trig functions given information about the unit circle, or the reference angle.
   - How to simplify trigonemetric expressions.
   - How to simplify expressions involving exponentials and logarithms.
   - How to complete the square to rewrite a quadratic function, and how to factor a quadratic expression.

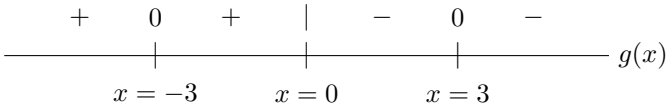
Here are some sample questions similar to what you should expect to see on the exam.

1. Calculate the average rate of change of $a(x) = \frac{2}{x} + 1$ on the generic interval $[x, x+h]$.
2. Calculate the average rate of change of $b(x) = 2\cos(x)$ on the interval $[0, \pi/3]$.
3. Calculate the average rate of change of $c(x) = x^3 - 2x^2 + 1$ on the generic interval $[x, x+h]$.
4. Consider the following graph of the function $F(x)$.



- What is the average rate of change of $F(x)$ over the interval $[-9, 0]$?
- Is the slope of the tangent line to the graph at $x = -3$ positive, negative, or zero?
- What about at $x = -5$?
- If you compare these two tangent lines, which one's slope is greater?
- On the interval $(-\infty, -1)$ the function $F(x)$ is which: increasing or decreasing? At an increasing or decreasing rate?
- What about on the interval $(-1, \infty)$.

5. Find all x- and y-intercepts of $a(t) = t^2 - 4t - 12$.
6. Find all x- and y-intercepts of $b(t) = 1 - e^{2+t}$.
7. Find all x- and y-intercepts of $c(t) = 2(t+1)^3 - 1$.
8. Find all x- and y-intercepts of $d(t) = \sqrt{2-t} - 4$.
9. If $f(x) = x^2 + 4x - 3$ find all inputs x with an output of $f(x) = 2$.
10. Find the general solution to $2\cos x + \sqrt{2} = 0$.
11. Find the general solution to $3\sin x + 2 = 4$.
12. Find the general solution to $\tan(3x) + 1 = 0$.
13. Find all x-intercepts of $p(x) = -2x^3(x+4)(x-2)^2(x-10)$.
14. Find all x- and y-intercepts of
$$q(x) = \frac{4x^5 + 2x^4 - x^3}{x^2 + 2x + 1}.$$
15. Determine the domain of
$$f(x) = \ln\left(\frac{x^5 - 4x^4 + 4x^3}{x^2 + 2x + 1}\right)$$
Give your answer in interval notation.
16. Solve the inequality $-2(x-1)(x+3)^2(x-3)^4 \geq 0$. Give your answer in interval notation.
17. Suppose you know that a rational function $g(x)$ has leading term $\frac{-2x^6}{x^9}$ and has the following sign diagram. Use this information to solve the inequality $g(x) \geq 0$.



18. Determine the domain of $g(x) = 4x^3 - 3x^2 + 2x + 10$.
19. Determine the domain and range of $h(x) = -(x+2)^2 + 1$. Where is the function increasing? Where is it decreasing? Is it concave up or concave down?
20. What are the domain and range of $j(x) = 1 - 2^{x-1}$? Is the function increasing or decreasing? Concave up or concave down?
21. What is the domain of $k(x) = \sqrt[4]{x-1} + 3$?
22. What are the domain and range of $\ell(x) = -\log_{1000}(5-x)$?
23. Sketch a graph of one full period of $w(x) = -3\cos(2x)$. What are the amplitude and period of the wave?

24. A wave oscillates between a minimum of -2 and a maximum of 6 with a period of 4. If the wave starts at the maximum at time 0, write a trig function which models the wave.

25. Find the vertex and any zeroes of $q(s) = -s^2 + 2s - 4$, and sketch a graph of q.

26. Identify what geometric transformations were applied to a basic power function x^n to get $f(x) = -3(x+1)^5 - 3$. Find the intercepts of $f(x)$ and sketch a graph, identifying the intercepts and inflection point.

27. Sketch a graph of $g(x) = 3 \cdot 2^{x-1}$, identifying all asymptotes. What are the domain and range of $g(x)$?

28. Sketch a graph of $h(x) = \log_3((2x-3)^2)$, identifying all asymptotes. What are the domain and range of $h(x)$?

29. Sketch a graph of the following piecewise-defined function:

$$\ell(x) = \begin{cases} 2x - 1 & x < 1 \\ 1 & x \geq 1 \end{cases}$$

30. Give an angle θ so that $\theta = \arcsin(\sin(\theta))$.

31. Let $p(x) = x^2 + 2x$ and $q(x) = \frac{1}{x}$. Compute $(pq)(x)$, $p(q(x))$, and $q(p(x))$. Fully simplify your answers.

32. Find the inverse of $b(x) = 3(x-1)^3 + 8$.

33. Find the inverse of $c(x) = e^{2x-2} + 3$.

34. Find the inverse of $d(x) = -\log_2(3x) - 3$.

35. Find the inverse of $f(x) = \sqrt{3x+1} - 1$. What is the domain of $f^{-1}(x)$?

36. Give the exact values for:

$\cos(0)$	$\sin(\pi)$	$\tan(\pi/2)$
$\csc(\pi/4)$	$\sec(-2\pi/3)$	$\cot(\pi/4)$
$\arcsin(-1)$	$\arccos(\sqrt{2}/2)$	$\arctan(1)$

37. The angle θ is in quadrant 2 and has reference angle α. If $\cos(\alpha) = 3/5$, determine $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.

38. Fully simplify the trig expression

$$\frac{\sin x}{\csc x} + \cot x \sin x.$$

39. Rewrite the quadratic expression $2x^2 - 10x + 6$ in factored form.

40. Rewrite the quadratic expression $2x^2 - 10x + 6$ in vertex form.

41. Fully simplify the following expression. All exponents should be positive.

$$(2x)^3 \cdot (xy)^{-5}$$

42. Use rules for logarithms to rewrite this expression to have a single log:

$$\log_3(x-1) + \log_3(x+1) - 3\log_2(x).$$

43. Fully simplify the following expression. Your final answer should not contain any logs.

$$\log_2\left(\frac{8^2}{4}\right)$$

44. Fully simplify the following expression. Your final answer should contain a single exponential.

$$\left(e^{2x}\right)^3 \cdot e^3$$