Math 1420: Study Guide for Midterm 2

General comments:

- The format for the exam is similar to the previous exam. You should expect roughly the same number of questions, again broken up by the four learning objectives.
- Calculators and notes are not allowed for the exam. The questions are written with the fact that these are not allowed in mind. In particular, you should expect that any numbers involved in calculations will be relatively small and manageable.
- In particular, you will not be given a formula sheet for the rules for derivatives. A big part of the exam is whether you remember and understand those rules!
- Show your work! For one, understanding the process and how to communicate your logic to others is more important than being able to produce a correct answer with no explanation. For another, I cannot give partial credit if you show no work.

Here's what you should know for each learning objective.

1. Conceptual Understanding
   - The relationship between derivatives and tangent lines, and what derivatives tell you about whether a function is increasing or decreasing, and concave up or concave down.
   - The concept behind implicit differentiation.
2. Formal Understanding
   - How to do logarithmic differentiation.
   - How to derive a rule for differentiation from other rules for differentiation
3. Rules for Calculations
   - The rules for calculating derivatives, including: the product rule, the quotient rule, the chain rule, the power rule, and the rules for exponential functions, logarithms, trig functions, and inverse trig functions.
   - For inverse trig functions, you are only expected to know the rules for arcsin, arccos, and arctan.
4. Approximations and Applications
   - How to use implicit differentation to calculate slope of a curve.
   - How to solve related rates problems.

Here's what you should focus on:

- Know the rules for differentiation!
- Know how to do implicit differentiation and logarithmic differentiation.

Here are some sample questions similar to what you should expect to see on the exam.

1. Suppose you know that the tangent line to the graph of $f(x)$ at $x = 2$ is given by the equation $y = 3x - 1$. What are $f(2)$ and $f'(2)$?

2. Suppose you know that $f'(x) = (x-2)^3$. Using this information, determine on what interval(s) $f(x)$ is increasing, and on what interval(s) $f(x)$ is decreasing.

3. Suppose you know that $f''(x) = 3(x-2)^2$. Using this information, determine on what interval(s) $f(x)$ is concave up, and on what interval(s) $f(x)$ is concave down.

4. You and your friend are trying to understand the curve described by the equation $e^{xy} = y^4 + x^2$. As part of this, you need to figure out the slope of the curve at the point $(1, 0)$. Your friend insists that you need to solve for y as a function of x and then differentiate to find the slope. Is your friend correct? If so, follow their method to find the slope. If not, explain the alternate method you would use.

5. State the product rule, then use the other rules for derivatives to derive it.

6. State the quotient rule, then use the other rules for derivatives to derive it.

7. Use logarithmic differentiation to differentiate $a(t) = t^{(t^2)}$. Don't bother to fully simplify your answer.

8. Use logarithmic differentiation to differentiate

$$b(t) = \frac{e^{7t}t^3}{\sin t}.$$

Don't bother to fully simplify your answer.

9. Consider $c(x) = e^x$. Find $c'(x)$.

10. Consider $d(x) = x^7 - 5x^5 + 2x^4 - 2x^3 - 7$. Compute $d'(x)$ and $d''(x)$.

11. Consider $f(x) = \sin(2x) \cdot \cos(x)$. Compute $f'(x)$.

12. Consider

$$g(x) = \frac{e^x + x}{x^2 + 1}.$$

Compute $g'(x)$.

13. Consider $h(x) = 2\arctan(\sqrt{x})$. Find $h'(x)$.

14. Consider $j(x) = \log_2(x^2 - 1)$. Find $j'(x)$.

15. Consider $k(x) = 4^{2x} - \arcsin(2x)$. Find $k'(x)$.

16. Consider $\ell(x) = \ln(\tan x)$. Find $\ell'(x)$.

17. Consider $m(x) = \arccos(\csc 2x)$. Find $m'(x)$.

18. The equation $e^{xy} = x$ defines a curve. Find the slope of the curve at the point $(e, 1/e)$.

19. The equation $3x^2+xy+2y^2 = 10$ defines an ellipse. Find the slope of the ellipse at the points $(1, \frac{3\sqrt{3}-1}{2})$ and $(1, -\frac{3\sqrt{3}+1}{2})$.

20. The equation $2x^2-4xy+y^2 = 8$ defines a hyperbola. Find the slope of the hyperbola at its x-intercepts.

21. You are using 3d animation software to create an animation. As part of this animation, a sphere is shrinking in size. You have the software set up so that the radius of the sphere is decreasing at a constant rate of 12 units per second. When the radius is 120 units, what is the rate of change of the volume of the sphere? [Hint: The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.]

22. You have successfully leveraged your undergraduate degree in pure mathematics to get a job as a barista. As part of this job you are preparing a pour-over coffee for a customer, and your mind wanders back to the calculus I class you took. The filter you are using is conical, with a height of 2 inches and a diameter of 2 inches. You pour hot water into the filter at a rate of 1 cubic inch per second. At what rate is the depth of water in the filter increasing when you've filled it to its maximum height? [Hint: The formula for the volume of a cone is $V = \frac{\pi}{3}r^2h$.]