Math 1420: Study Guide for Midterm 3

General comments:

- The format for the exam is similar to the previous exams. You should expect roughly the same number of questions, again broken up by the four learning objectives.
- Calculators and notes are not allowed for the exam. The questions are written with the fact that these are not allowed in mind. In particular, you should expect that any numbers involved in calculations will be relatively small and manageable.
- Show your work! For one, understanding the process and how to communicate your logic to others is more important than being able to produce a correct answer with no explanation. For another, I cannot give partial credit if you show no work.

Here's what you should know for each learning objective.

1. Conceptual Understanding
   - The meaning of the mean value theorem, Rolles's theorem, and the extreme value theorem.
   - What conditions need to be checked for each of these theorems to apply.
2. Formal Understanding
   - How to find absolute maximums and minimums for a continuous function on a closed and bounded interval.
   - How to use the first and second derivative tests to determine where a function has local maximums and minimums.
   - How to find inflection points of a function, and how to determine where a function is increasing, decreasing, concave up, and concave down.
3. Rules for Calculations
   - How to use L'Hôpital's rule to compute limits in the indeterminate form $0/0$, ∞/∞, $0 \times \infty$, and $\infty - \infty$. (I will not ask you about the indeterminate forms 1^∞ and ∞^0, for which you need to take logarithms.)
   - How to compute antiderivatives of power functions, exponential functions, and sin and cos, or weighted sums of these functions.
4. Approximations and Applications
   - How to set up and solve optimization problems.

Here are some sample questions similar to what you should expect to see on the exam.

1. Your friend tells you that the average rate of change of $f(x) = 1/x$ on the interval $[-1, 1]$ is 1, and claims that according to the mean value theorem this means there's a point c where $f'(c) = 1$. Is your friend right? Explain why, or if they're wrong explain the flaw in their reasoning.
2. Does the function $g(x) = x + \frac{\sin(2x-\pi)+1}{x+4}$ have an absolute maximum and minimum on the interval $[0, 100]$? Explain why or why not without actually calculating any extreme points.

3. Consider the function $h(x) = x^2 - 4x + 1$. Can you use Rolles' theorem to know that there is *no* point c between 0 and 5 where $h'(c) = 0$? Justify your answer with a short explanation.

4. Find the absolute maximum and minimum of the function $a(x) = \frac{1}{x} + x$ on the interval $[1/2, 5/2]$.

5. Find all local maximums and minimums of the function $b(x) = x^3 - 2x^2 + x$. Justify your answer of which critical points are maximums versus minimums.

6. Find all local maximums and minimums of the function $c(x) = x^3 e^{3x}$.

7. Consider the function $d(x) = \ln x - 2x^2$. Determine on what intervals $d(x)$ is increasing, decreasing, concave up, and concave down.

8. Use L'Hôpital's rule to compute the limit
$$\lim_{x \to 0} \frac{\cos x - 1}{x}.$$

9. Use L'Hôpital's rule to compute the limit
$$\lim_{x \to \infty} \frac{e^{x/100}}{50x^2}.$$

10. L'Hôpital's rule to compute the limit
$$\lim_{x \to 0} \cot x - \frac{1}{x}.$$

11. If $f(x) = x^3 + 3x^2$, determine its antiderivative $F(x)$. [Hint: Don't forget the $+C$!]

12. If $g(x) = \sin x + 3e^x$, determine its antiderivative $G(x)$. [Hint: Don't forget the $+C$!]

13. Find the point on the line $y = 3x - 2$ which minimizes the distance to the point $(2, 5)$.

14. You are building a cylindrical container whose base and top are perfect circles. You have a budget of 72 dollars for materials. If the material for the base costs 4 dollars per square meter, the material for the top costs 2 dollars per square meter, and the material for the sides costs 1 dollar per square meter, which is the largest volume you can enclose?