MATH 1420: WORKSHEET FOR SECTION 4.5
LOCAL MAXES AND MINS

We learned two tests for finding local maximums and minimums of a function $f(x)$.

- For both tests, the first step is to find the critical points of $f(x)$. These are where $f'(x) = 0$ or $f'(x)$ is undefined.

What you do next depends upon whether you're using the **first derivative test** or the **second derivative test**.

The first derivative test is about the sign diagram of $f'(x)$.

- **First derivative test.**
- The next step is to fill out the sign diagram for $f'(x)$, determining where it is positive or negative. By identifying the critical points you've already found where it could change sign, so all that remains is to determine what happens at the regions between. For this, you only have to look at one point in the region—all points in the region have the same sign.
- Now look at the sign diagram around each critical point. Positive to the left and negative to the right means you have a maximum. Negative to the left and positive to the right means you have a minimum. Both sides having the same sign means its neither a max nor a min.

The second derivative test is about the sign of $f''(x)$ at the critical points.

- **Second derivative test.**
- This only works for the critical points c where $f'(c) = 0$. If $f'(c)$ is undefined, you cannot use this test.
- The next step is to compute $f''(x)$.
- Then calculate $f''(c)$ at each critical point c.
- If $f''(c)$ is positive, you have a minimum. If $f''(c)$ is negative, you have a maximum. If $f''(c) = 0$, it is neither a max nor a min.

Here are some problems to test your abilities with these tests.

(1) Find all local maximums and minimums of $a(x) = \frac{x^3}{3} + 4x^2 - 10$.
(2) Find all local maximums and minimums of $b(x) = x \ln x$.
(3) Find all local maximums and minimums of

$$c(x) = \frac{e^x}{x}.$$

(4) Find all local maximums and minimums of $d(x) = \sqrt{x^2 + 4}$.
(5) The height of a rock thrown upward is modeled by $y(t) = 2 + 20t - 5t^2$, where $y(t)$ is the height in meters of the rock t seconds after being thrown. Determine when the rock reaches its maximum height, and what that height is.
(6) You have 500 feet worth of fencing to enclose a pasture, bordered on one side by a perfectly straight stream. You want to build a rectangular pasture, with the stream providing one of the four sides of the stream.
   (a) Write a function $A(x)$ which gives the area of the pasture if the length of fence parallel the stream is x feet.
   (b) Find the maximum of $A(x)$ to determine the largest area you could enclose with that amount of fencing.