MATH 1420: WORKSHEET FOR SECTION 5.3
THE FUNDAMENTAL THEOREM OF CALCULUS

The Fundamental Theorem of Calculus is usually presented in two parts. Let's start with what the book makes part 2.

The Fundamental Theorem of Calculus, part 2. Consider $f(x)$ a function continuous on the interval $[a, b]$ and let $F(x)$ be any antiderivative of $f(x)$. Then,

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a).$$

This theorem says you can evalate definite integrals using antiderivatives. No need to do a Riemann sum or take some complicated limit. Just plug in the values of an antiderivatives at the endpoints!

Part 1 is less directly useful, but it explains where part 2 comes from.

The Fundamental Theorem of Calculus, part 1. Consider $f(x)$ a function continuous on the interval $[a, b]$. Define a function $F(x)$ on $[a, b]$ as:

$$F(x) = \int_a^x f(t)\,\mathrm{d}t.$$

Then, $F(x)$ is an antiderivative of $f(x)$, i.e. $F'(x) = f(x)$.

Note that in the definition of $F(x)$, that the upper limit of the integral is a variable. So the total area changes based on what x is.

Evaluate the following definite integrals.

(1) $\int_0^3 x^2 - x \, \mathrm{d}x.$

(2) $\int_4^1 x^2 - x \, \mathrm{d}x.$

(3) $\int_0^\pi \sin x \, \mathrm{d}x.$

(4) $\int_1^4 \frac{2}{t} \, \mathrm{d}t.$

(5) $\int_{-10}^{10} e^y \, \mathrm{d}y.$

For each of the following functions defined by integration, determine their derivatives.

(1) $f(x) = \int_0^x e^{-t^2} \, \mathrm{d}t.$

(2) $g(x) = \int_0^{\sqrt{x}} e^{-t^2} \, \mathrm{d}t.$

(3) $h(x) = \int_x^{2x} e^{-t^2} \, \mathrm{d}t.$

(4) $j(x) = \int_{\sin x}^0 \cos(x) \, \mathrm{d}x.$