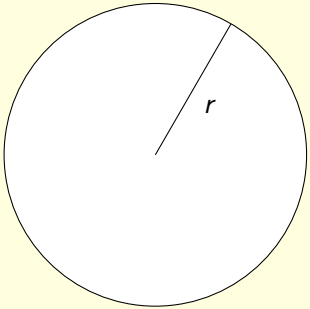
Math 210: Calculus I

Kameryn J Williams

Bard College at Simon's Rock

Fall 2023

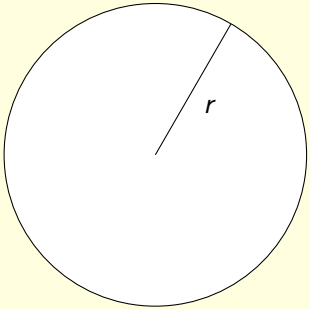
Circles



$$\text{Circumference} = 2\pi r$$

$$\text{Area} = \pi r^2$$

Circles



The circumference formula is the definition of π; the circumference is a ratio of the radius, and 2π is the number that gives that ratio.

But where does the area formula come from?

$$\text{Circumference} = 2\pi r$$

$$\text{Area} = \pi r^2$$

The area of a circle

Sato Moshun's calculation of the area of the circle, 17th century

The big idea of calculus

Looking at what happens when a quantity becomes infinitely small or two quantities get infinitely close to each other gives you a lot of information.

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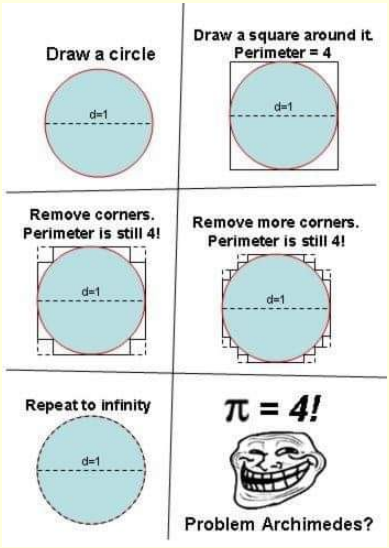
Big Concepts

- Limit: What happens when a quantity gets infinitely close to another
- Derivative: Instantaneous rate of change or slope of a function
- Integral: Area under a curve

But we have to be careful!



A brief history of the calculus

- (Before 17th century) Many different mathematicians around the globe (such as Archimedes of Syracuse and Madhava of Sangamagrama) use infinitesimals to make calculations—volumes, areas, lengths.
- (17th–18th centuries) Gottfried Wilhelm Leibniz and Isaac Newton independently invent a systematic framework for this species of calculation, and prove the fundamental theorem of calculus which explains the connection between derivatives and integrals. Later mathematicians build on their work and the modern discipline of calculus is born.

  But this framework faced criticism for its use of infinitesimals.
- (19th century) Karl Weierstrass and other mathematicians show how to redo calculus in a framework that doesn't use infinitesimals.
- (1960s) Abraham Robinson gives a mathematically rigorous framework for calculus using infinitesimals.