MATH 210: 10-13 WORKSHEET

A function $y = f(x)$ might be hard to differentiate, but the trick of *logarithmic differentiation* can be used.

- Take ln of both sides: $\ln y = \ln(f(x))$. This equation describes the same curve because ln is one-to-one.
- Use rules for logarithms to rewrite the righthand side:

$$\ln(ab) = \ln a + \ln b, \qquad \ln(a/b) = \ln a - \ln b, \qquad \ln(a^b) = b \ln a.$$

- Do implicit differentiation on the equation.
- Solve for y' in terms of x and y.
- Replace y with $f(x)$ to get $f'(x) = y'$ as a function of x.

(1) Use logarithmic differentiation to differentiate $a(x) = x^x$. Why can you use neither the power rule nor the exponential function rule here?
(2) Differentiate the function $b(t) = t^2 e^t \sin t$ by using the product rule twice.
(3) Now use logarithmic differentiation to find $b'(t)$. Which of the two methods was faster?
(4) Use logarithmic differentiation to differentiate $c(u) = \frac{u^2+1}{e^u}$.
(5) Use logarithmic differentiation to differentiate $d(y) = (y^2 - 4)^{2 \sin y}$.
(6) Use logarithmic differentiation to derive the power rule: if $f(x) = x^a$, where $a \neq 0$ is constant, then $f'(x) = ax^{a-1}$.
(7) Differentiate $g(t) = (\tan t)^\pi$.