MATH 210: 10-23 WORKSHEET

The **Extreme Value Theorem** gives you a method for finding the *absolute minimum* and *absolute minimum* of a function $f(x)$ on a closed bounded interval $[a, b]$:

- Get a formula for $f'(x)$.
- Solve $f'(x) = 0$ and determine where $f'(x)$ is undefined. You only care about answers in the interval $[a, b]$. These are the *critical points* of $f(x)$.
- Compute the output $f(x)$ of the original function at each of the critical points, plus at $x = a$ and $x = b$.
- The EVT tells you that $f(x)$ does have an absolute maximum and minimum. The largest value from the previous step is the maximum, the smallest value is the minimum.

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(1) Use a computer tool to graph $a(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x - 3$ on the interval $[-3, 3]$. [If you use `desmos.com`, you can type `1/3x^3 + 1/2x^2 - 2x - 3 {-3 <= x <= 3}` to restrict the domain.] Look at the graph to visually identify the critical points. Where is the absolute maximum? Where is the absolute minimum?

(2) Confirm your work by calculating $a'(x)$, solving for the critical points, and finding the values, as via the algorithm described above.

(3) Find the absolute maximum and minimum of $c(x) = \sqrt[3]{x^2}$ on the interval $[-1, 8]$. Check your answers by using a computer tool to graph $c(x)$.

(4) Check that all the hypotheses in the extreme value theorem are necessary. Specifically, give graphs for:
- A function on $[a, b]$ which is not continuous and has no absolute maximum nor minimum in the interval.
- A function continuous on an unbounded interval $[a, \infty)$ or $(-\infty, b]$ or $(-\infty, \infty)$ which has no absolute maximum in the interval. Can you also get no absolute minimum? Can you get neither?
- A function continuous on an open interval (a, b) which has no absolute maximum nor minimum in the interval.

(5) You are fencing in a rectangular area, with the side of your house forming one side of the rectangle, for an enclosed space for your dog to run around in. To give her plenty of space, the area must be 800 square feet. Determine the minimum length of fencing you need to enclose this space.
- (a) First draw and label a picture describing this setup.
- (b) Write a formula that tells you the total length of fencing $f(x, y)$ in terms of the two side lengths x and y of the rectangle.
- (c) Use that the area must be 800 to solve for x in terms of y. Then substitute this into your original formula to get only one variable—so $f(x)$.
- (d) Differentiate $f(x)$ and find its critical points.
- (e) Calculate the value of $f(x)$ at the critical points and use this to determine the minimum length of fencing needed. How long should the sides of the rectangle be to acheive this minimum length?
- (f) Check your work by graphing $f(x)$ and looking for the minimum.