MATH 210: 10-27 WORKSHEET

The first and second derivative tell you about the shape of a graph:

- $f'(x) > 0$ means $f(x)$ is increasing, and $f'(x) < 0$ means $f(x)$ is decreasing;
- $f''(x) > 0$ means $f(x)$ is concave up, and $f''(x) < 0$ means $f(x)$ is concave down.
- *Local minimums* occur where $f'(x)$ changes from negative to positive, equivalently at critical points where the graph is concave up.
- *Local maximums* occur where $f'(x)$ changes from positive to negative, equivalently at critical points where the graph is concave down.
- *Inflection points* are where the concavity changes, equivalently where $f''(x)$ goes from negative to positive or vice versa.

Because of these facts, a *sign diagram* for $f'(x)$ and $f''(x)$ give you all the information you need to find local maximums/minimums and inflection points.

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(1) Use computer tools to graph $a(x) = x^2e^x$. Look at the graph to visually determine all local maximums and minimums. Try to guess approximately where any inflection points are.

(2) Confirm your work by computing $a'(x)$ and $a''(x)$, solving when they are 0, and creating the sign diagrams for the functions. Use the sign diagrams to determine where the local extremums are and inflection points are.

(3) Use computer tools to graph $b(t) = \sin t$. Look at the graph to visually determine where the local extremums and inflection points are. Explain the meaning of these as they relate to the unit circle.

(4) Calculate $b'(t)$ and $b''(t)$. What equations would you need to solve to find the critical points and inflection points of $b(t)$?

(5) Is it possible for a critical point c to be both a local minimum and an inflection point for a differentiable function $c(x)$? If yes, give an example. If no, explain why.