MATH 210: 10-30 WORKSHEET
ANTIDERIVATIVES

If $f(x)$ is a function, an *antiderivative* of $f(x)$ is a function $F(x)$ so that $F'(x) = f(x)$.

(1) Check that a function can have more than one antiderivative by checking that $A_1(x) = x^3 + 3$ and $A_2(x) = x^3 - 4$ are both antiderivatives of $a(x) = 3x^2$.

(2) As a corollary of the mean value theorem, we saw that two functions have the same derivative if and only if they differ by a constant. Thus, the *general antiderivative* of a continuous function $f(x)$ is $F(x) + C$, where $F(x)$ is any antiderivative and C is a parameter standing for an arbitrary constant. Varying the value of C gives all the different antiderivatives of $f(x)$.
Find the general antiderivative of $a(x) = 3x^2$.

Hereon, if $f(x)$ or $g(x)$ or ... denotes a function let $F(x)$ or $G(x)$ or ... denote an antiderivative. That is, let's denote an antiderivative by making the letter capital. Also assume that all functions are continuous. (We will later see that any continuous function has an antiderivative.)

To compute antiderivatives, your most basic tool is the atomic rules for derivatives, but backward. First, however, let's check that the sum and constant multiplication rules for derivatives can be done backward.

(1) Confirm that $F(x) + G(x)$ is an antiderivative of $f(x) + g(x)$. What does the general antiderivative look like?

(2) For a constant c, confirm that $c \cdot F(x)$ is an antiderivative of $c \cdot f(x)$. What does the general antiderivative look like?

Now let's look at atomic rules.

(1) Figure out a backward version of the power rule. First, determine an antiderivative of $(\alpha + 1)x^\alpha$, where $\alpha \neq -1$ is a constant. Then use this to determine the general antiderivative of x^α.

(2) For the backward power rule, we need that the exponent α is not -1. Explain why we did this by looking at the atomic rules for derivatives to determine an antiderivative of $\frac{1}{x}$.

(3) Look at the atomic rules for derivatives to determine an antiderivative of e^x.

(4) Look at the atomic rules for derivatives to determine an antiderivative of $\cos x$.

(5) Look at the atomic rules for derivatives to determine an antiderivative of $\sin x$. [Hint: it might be easier to first determine an antiderivative of $-\sin x$.]

(continued on back)

Now let's put this into practice. Determine the general antiderivative for each of the following functions by using backward versions of the atomic rules for derivatives and the sum and constant multiple rules. Check your answers by differentiating them.

(1) $a(x) = 2x + x^2$
(2) $b(x) = \frac{3}{\sqrt{x}}$
(3) $c(x) = 4e^x - \frac{1}{x}$
(4) $d(x) = \sin x + \cos x$
(5) $f(x) = \pi \sec^2 x - \csc^2 x$
(6) $g(x) = 17$

Finally, I want to leave you with a warning: computing antiderivatives is generally harder than computing derivatives. Very often there isn't an atomic rule you can run backward!

(1) Try to determine an antiderivative of $\ln x$ by looking at the atomic rules for derivatives.
(2) After you give up at the previous problem, compute the derivative of $x \ln x - x$. Use your answer to this to determine the general antiderivative of $\ln x$.
    [If this looks totally out of nowhere: in Calc 2 you will learn a method for finding antiderivatives (namely, *integration by parts*) that could be used to determine the general antiderivative of $\ln x$.]