MATH 210: 11-1 WORKSHEET
SUMS

We learned new notation today for expressing sums:

$$\sum_{i=1}^{N} f(i) = f(1) + f(2) + f(3) + \cdots + f(N).$$

This *summation notation* or *sigma notation* is a compact way of expressing the sum of the *summands* $f(1)$, $f(2)$, etc.

Your index variable i doesn't start at 1. Here's an example of this:

$$\sum_{i=7}^{13} \sqrt{i} = \sqrt{7} + \sqrt{8} + \cdots + \sqrt{13}.$$

Note that you could *re-index* this sum to start at $i = 1$ by adjusting the summands:

$$\sum_{i=1}^{7} \sqrt{i+6} = \sqrt{7} + \sqrt{8} + \cdots + \sqrt{13}.$$

You can similarly reindex to start at any integer. Try rewriting this sum to start at $i = 0$. Then try rewriting to start at $i = 14$.

If you have some programming experience, it might be helpful to see an example of some code to compute a sum. The sum

$$\sum_{i=1}^{100} 2i + 1$$

is computed by the code snippet

```
sum = 0;
for (int i = 1; i <= 100; i++)
    sum += 2*i + 1;
return sum;
```

(1) Write out the expansion of the sum

$$\sum_{i=1}^{5} i^2$$

then compute the sum.

(2) Write out the expansion for the sum

$$\sum_{k=10}^{12} 2k - 1$$

then compute the sum.

(3) Consider the sum

$$2 + 3 + 4 + \cdots + 11.$$

Write this sum in sigma notation. Do this twice, first with the index variable starting at 2, then starting at 0.

(4) This notation also gets used where there isn't a natural order to the terms in the sum. For example, the *mean* of a finite set of numbers is their sum divided by the size of the set. In symbols:

$$\bar{x} = \frac{\sum_i x_i}{n},$$

says that $\bar{x}$ is the mean of the n many numbers x_i.
Compute the mean of the set $\{0, 4, 12, 20\}$. Why does it not matter the order in which you add up the numbers?

(5) You decide to approximate the area under the curve $f(x) = 9 - x^2$ along the interval $[0, 3]$ by splitting the interval into three equal pieces and using a rectangle to approximate the area for each piece. To do this you have to decide the height of each rectangle.
   (a) Use the value of the function at the left end of each rectangle for the heights. Sketch a graph of $f(x)$ and draw the three rectangles on the graph. Write this sum in sigma notation and compute its value.
   (b) Use the value of the function at the right end of each rectangle for the heights. Sketch a graph of $f(x)$ and draw the three rectangles on the graph. Write this sum in sigma notation and compute its value.
   (c) Use the value of the function at the middle of each rectangle for the heights. Sketch a graph of $f(x)$ and draw the three rectangles on the graph. Write this sum in sigma notation and compute its value.