MATH 210: 11-10 WORKSHEET

(1) Recall that an *even function* is one whose graph is symmetric across the y-axis. In symbols, $f(x)$ is even if and only if $f(-x) = f(x)$ for all x. If a function is even then this symmetry can be used to simplify integrals. Graph $\cos x$ and explain why

$$\int_{-\pi/2}^{\pi/2} \cos x \, \mathrm{d}x = 2 \cdot \int_{0}^{\pi/2} \cos x \, \mathrm{d}x,$$

then compute that integral.

(2) It looks really hard to calculate the integral

$$\int_{-1}^{1} \sin(x^3 - x) \, \mathrm{d}x.$$

Use computer tools to graph $\sin(x^3 - x)$. What do you notice about the graph? Why does this observation let you conclude that the integral $= 0$? Can you generalize this observation?

(3) Use the FTC part II to evaluate

$$\int_{0}^{3} 4x - 3x^2 \, \mathrm{d}x.$$

(4) Use the FTC part II to evaluate

$$\int_{1}^{4} \sqrt{x} - \sqrt[3]{x} \, \mathrm{d}x.$$

(5) Evaluate

$$\int_{1}^{e} \frac{\mathrm{d}x}{x}.$$

(6) Use the FTC part II to evaluate

$$\int_{0}^{6} x^2 \, \mathrm{d}x.$$

Then approximate this integral by computing the left and right Riemman sums with $N = 3$ pieces. Which gives a better approximation? Take the average of the two sums. Is this a good approximation? Time permitting, compute the left and right Riemann sums for $N = 6$, along with their average, and compare those to the true area given by the FTC part II.

An *indefinite integral* is like a definite integral but without having the endpoints a and b determined. Because the FTC part II says you compute definite integrals via an antiderivative, the indefinite integral is defined to be the general antiderivative. In symbols:

$$\int f(x)\,\mathrm{d}x = F(x) + C,$$

where $F(x)$ is an antiderivative of $f(x)$ and C is an arbitrary constant.

(1) Compute
$$\int e^x - 2^e e^x\,\mathrm{d}x.$$

(2) Compute
$$\int x^4 - \frac{1}{\sqrt[4]{x}}\,\mathrm{d}x.$$

(3) Compute
$$\int x^2 - 4x + 1\,\mathrm{d}x.$$

(4) Using what you just did, calculate
$$\int_0^1 x^2 - 4x + 1\,\mathrm{d}x \qquad \text{and} \qquad \int_{-1}^3 x^2 - 4x + 1\,\mathrm{d}x.$$