MATH 210: 11-6 WORKSHEET

We don't yet have the Fundamental Theorem of Calculus, which provides the main tool for calculatign definite integrals. But knowing that integrals are about area you can still calculate some integrals.

(1) Calculate
$$\int_{-1}^{7} 3\,\mathrm{d}x,$$
by using the fact that it's giving the area of a rectangle. More generally, what is
$$\int_{a}^{b} c\,\mathrm{d}x$$
if c is a constant?

(2) A region under a linear function is composed of triangles or trapezoids. Using the area formulas for these shapes, calculate:
$$\int_{0}^{2} 6-2x\,\mathrm{d}x$$
$$\int_{0}^{3} 6-2x\,\mathrm{d}x$$
$$\int_{0}^{4} 6-2x\,\mathrm{d}x$$

(3) The equation $y=\sqrt{r^2-x^2}$ describes the top half of the circle with radius r centered at the origin. Determine
$$\int_{0}^{4} \sqrt{16-x^2}\,\mathrm{d}x.$$

(4) The *average value* of $f(x)$ over the interval $[a,b]$ is the value y so that the rectangle on $[a,b]$ with height y has the same area as under $f(x)$. In symbols:
$$\text{average value} = \frac{1}{b-a}\int_{a}^{b} f(x)\,\mathrm{d}x.$$
Determine the average value y of $f(x)=x+3$ over the interval $[1,5]$. Once you determine y, graph both $f(x)$ and the constant function with value y.

(5) What is the average value of $\sqrt{1-x^2}$ over the interval $[-1,1]$?