MATH 210: 12-4 WORKSHEET
MORE PRACTICE WITH SUBSTITUTION

When we learned the chain rule for derivatives, we saw that you could apply it repeatedly for a function given by repeated composition. When you do substitution—the backward version of the chain rule—you only need to do it once.

(1) Differentiate $\tan(e^{2x^3+1})$. How many times did you have to apply the chain rule?

(2) Integrate
$$\int 2xe^{x^2}\cos(e^{x^2})\,\mathrm{d}x$$
by using the substitution $u = e^{x^2}$.

(3) Integrate
$$\int -\frac{e^{\cos(\sqrt{x})}\sin(\sqrt{x})}{2\sqrt{x}}.$$

Sometimes it's not obvious you can use substitution, and you first have to rewrite an integrand.

(1) Evaluate $\int \tan x\,\mathrm{d}x$ by rewriting the integrand as $\frac{\sin x}{\cos x}$.

(2) What is $\int \cot x\,\mathrm{d}x$?

(3) Evaluate $\int \sin^2 x \cdot \cos^3 x\,\mathrm{d}x$ by using the Pythagorean identity to rewrite $\cos^3 x = \cos^2 x \cdot \cos x = (1-\sin^2 x)\cos x$.

(4) What is $\int \sin^3\theta \cdot \cos^{1000}\theta\,\mathrm{d}\theta$?

Here are some more integrals where you need to do a small amount of rewriting to do substitution.

(1) Determine $\int \frac{\mathrm{d}x}{1+x^2}$ by looking at the rules for derivatives of inverse trig functions.

(2) Work out a rule for $\int \frac{\mathrm{d}x}{a^2+x^2}$, where a is a constant, by using the substitution $u = \frac{x}{a}$.

(3) Do a similar process to work out a rule for $\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}}$.

(4) Evaluate
$$\int \frac{2x+3}{4+x^2}\,\mathrm{d}x.$$
[Hint: Split it into two fractions.]

(5) What is
$$\int_0^2 \frac{2x+3}{4+x^2}\,\mathrm{d}x?$$