MATH 210: 12-6 WORKSHEET

Functions of the form $y(t) = y_0 e^{Kt}$, where $y_0 = y(0)$ is the initial value and K is a constant, have a lot of applications. This is because these are the functions which describe a quantity whose rate of change is proportional to its current size.

(1) Differentiate $y = e^{Kt}$. What does the second derivative look like? The third derivative? The nth derivative?
(2) Integrate $y = e^{Kt}$.

Sometimes people work with bases other than base e, but that is just a special case of base e, amounting to a specific choice for the constant K. A similar comment applies to logarithms.

(1) Use the change of base formula $b^t = e^{\ln(b)t}$ to differentiate b^t.
(2) Use this change of base formula to integrate b^t.
(3) Use the change of base formula $\log_b(t) = \frac{\ln t}{\ln b}$ to differentiate $\log_b(t)$.
(4) What is $\displaystyle\int \frac{\mathrm{d}t}{\ln(b)t}$?
(5) Check that $\displaystyle\int \ln x \,\mathrm{d}x = x(\ln x - 1) + C$ by differentiating the righthand side.
(6) Evaluate $\displaystyle\int \log_b x \,\mathrm{d}x$ by using first rewriting the integrand using the change of base formula for logarithms.

Now let's see an application.

(1) You are working in a biology lab, breeding fruit flies for research. You've found that the function $f(t) = 2e^{0.015t}$ gives a good approximation for the rate at which the fly population is increasing, in flies per day. If the initial population was 1000 flies, how many flies should you expect to have after 14 days?
(2) How long do you expect it to take for the population to double?