MATH 210: 9-11 WORKSHEET

For the following, either explain why the intermediate value theorem makes the statement true, or else produce a counterexample to show that the statement is false.

(1) If a function $f(x)$ has both positive and negative outputs then it has at least one zero.

(2) If $f(x)$ is a continuous function with $f(0) = f(10) = 1$ then $f(x)$ has no zeroes.

(3) If $f(x)$ is a continuous function with $f(-2) = 3$ and $f(2) = -2$ then $f(x)$ has exactly one zero between -2 and 2.

(4) If $f(x)$ is not continuous on $[0, 3]$ and $f(0) = 0$ and $f(3) = 2$ then there is no point x between 0 and 3 so that $f(x) = 1$.

(5) If the only discontinuities of $f(x)$ are removable discontinuities and $f(x)$ has both positive and negative outputs then it has at least one zero.