MATH 211: STUDY GUIDE FOR MIDTERM 2

Here are the big things you should know for the exam.

- How to do integration by parts.
- How to integrate functions involving powers of trig functions.
- How to do partial fraction decomposition, and how to use partial fraction decomposition to integrate a rational function.
- How to compute improper integrals.
- What it means for a sequence to converge and what it means for a series to converge.
- How to check whether a geometric series or a p-series converges.
- How to check for the convergence or divergence of series, using tests such as the ratio test, root test, direct comparison test, limit comparison test, or integral test.
- How to check whether an alternating series converges or diverges, and what it means for a series to converge absolutely versus converge conditionally.
- How to work out Maclaurin and Taylor series for functions based on important base functions like e^x, $\sin x$, or $\cos x$.

For this exam you will make and bring your own note sheet. This must be a standard 8.5" $\times$ 11" sheet of printer paper. You may use the front and back, and may either type it or write it by hand. You will put your name on it and turn it in with your exam.

Here's a few suggestions for what you might put on your note sheet:

- Trig identities used to compute integrals involving powers of trig functions.
- The basic integration rules that are used to compute integrals involving powers of trig functions.
- A reminder summary of when to use what method for integrals involving powers of trig functions.
- The basic integration rules that are used after doing partial fraction decomposition.
- A reminder summary of the steps for partial fraction decomposition.
- The tests for convergence or divergence of a series.
- The Maclaurin series for important functions.
- Any methods/rules for integration you want to reference.

Here's a list of sample problems.

(1) Evaluate $\displaystyle\int x^2 e^{4x}\,\mathrm{d}x$.

(2) Evaluate $\displaystyle\int \sin(x)\cdot\cos(2x)\,\mathrm{d}x$.

(3) Evaluate $\displaystyle\int_0^{\pi/9} 3x\sin(3x)\,\mathrm{d}x$.

(4) Evaluate $\displaystyle\int_0^{\pi/9} \ln x\,\mathrm{d}x$.

(5) Evaluate $\displaystyle\int \cos^4 x\sin^3 x\,\mathrm{d}x$.

(6) Evaluate $\int 3\sin^2 x \, \mathrm{d}x$.

(7) Evaluate $\int \sec^4 x \tan x \, \mathrm{d}x$.

(8) Evaluate $\int \csc^2 x \cot^{10} x \, \mathrm{d}x$.

(9) Evaluate $\int \sin^2(x) \, \mathrm{d}x$.

(10) Determine the partial fraction decomposition of

$$\frac{1}{x^2+4x}.$$

(11) Evaluate $\int \frac{1}{x^2+4x} \, \mathrm{d}x$.

(12) Determine the partial fraction decomposition of

$$\frac{2x-1}{x(x^2+4)}.$$

(13) Evaluate $\int \frac{2x-1}{x(x^2+4)} \, \mathrm{d}x$.

(14) Determine the partial fraction decomposition of

$$\frac{x^2+x}{x(x-1)^2}.$$

(15) Evaluate $\int \frac{x^2+x}{x(x-1)^2} \, \mathrm{d}x$.

(16) Evaluate $\int_0^{\infty} \frac{x^3}{x^4+4} \, \mathrm{d}x$.

(17) Evaluate $\int_0^{-\infty} e^x \sin x \, \mathrm{d}x$.

(18) Evaluate $\int_0^4 \frac{1}{\sqrt{x}} \, \mathrm{d}x$.

(19) Does this series converge or diverge? Explain why

$$\sum_{n=0}^{\infty} \frac{n^3}{n!}$$

(20) Does this series converge or diverge? Explain why

$$\sum_{n=0}^{\infty} \frac{n^3+4}{n^5-n}$$

(21) Does this series converge or diverge? Explain why

$$\sum_{n=0}^{\infty} \frac{(2n)!}{n^n}$$

(22) Does this series converge or diverge? Explain why

$$\sum_{n=0}^{\infty} \frac{n^3}{n!}$$

(23) Does this series converge absolutely, converge conditionally, or diverge? Explain why

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2^n}$$

(24) Does this series converge absolutely, converge conditionally, or diverge? Explain why

$$\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{n^2}$$

(25) You have a function $f(x)$ and you have computed that $f(2) = 1$, $f'(2) = 0$, $f''(2) = -1$, $f^{(3)}(2) = 3$, $f^{(4)}(2) = 1$. What is the degree 4 polynomial approximation to the Taylor series for $f(x)$ centered at $x = 2$?

(26) Determine the Taylor series centered at $x = 1$ for e^x. Give your answer in sigma notation.

(27) Determine the Maclaurin series for $\sin(x^3)$. Give your answer in sigma notation.