MATH 211: 10-27 WORKSHEET

If you think this class is your first time seeing infinite series, it isn't. Writing a number as an infinite sequence of digits is a shorthand for an infinite sum. For example,

$$\begin{aligned}\pi &= 3.14159265\ldots \\ &= 3 + \frac{1}{10} + \frac{4}{10^2} + \frac{1}{10^3} + \frac{5}{10^4} + \frac{9}{10^5} + \frac{2}{10^6} + \frac{6}{10^7} + \frac{5}{10^8} + \cdots\end{aligned}$$

(1) If the sequence of digits is one digit repeated forever, then this infinite series is a geometric series. Write the numbers $0.333\ldots$ and $0.999\ldots$ as geometric series, and use the formula for evaluating a geometric series to write them as fractions.

(2) If the sequence of digits is multiple digits repeated forever, then it's equivalent to a geometric series but you have to combine terms. For example,

$$0.252525\ldots = \frac{25}{100} + \frac{25}{100^2} + \frac{25}{100^3} + \cdots$$

Write $0.252525\ldots$ as a fraction.

(3) Show that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by comparing it to the integral $\int_1^{\infty} \frac{1}{x^2}\,\mathrm{d}x$, which we know converges by a calculation we did when looking at improper fractions. [Hint: Can you represent the infinite sum as the area of a bunch of rectangles so that you can compare that area to the area under the curve $\frac{1}{x^2}$?]

(4) Show that $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges to the integral $\int_1^{\infty} \frac{1}{x}\,\mathrm{d}x$, which we know diverges to ∞. [Hint: again think of the infinite sum as the area of infinitely many rectangles, but now you need to see they have larger area than the area under the curve $\frac{1}{x}$.]