MATH 211: 11-1 WORKSHEET

An *alternating series* is one in which the terms alternate between positive and negative.

- If a tail of the sequence $\{|a_n|\}$ monotonically decreases to a limit of 0, then $\sum_{n=0}^{\infty} a_n$ converges.

If a series $\sum_{n=0}^{\infty} a_n$ converges but the series $\sum_{n=0}^{\infty} |a_n|$ of absolute value diverges, we say it is *conditionally convergent*. If the sequence of absolute values converges then it is *absolutely convergent*.

- If a series is absolutely convergent then it converges.

For these series determine whether they diverge, converge absolutely, or converge conditionally.

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$

(2) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$

(3) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$

Consider the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$, where x is a fixed real number. Show that this series converges. [In fact, it converges absolutely. But we will need new tools to see why.]