MATH 211: 12-1 WORKSHEET

Use a computer tool to graph the following curves given by parametric equations.[1]

(1) $x(t) = t$, $y(t) = t^2$, $0 \leq t \leq 2$.
(2) $x(t) = \cos(3t)$, $y(t) = \sin(4t)$, $0 \leq t \leq 2\pi$.
(3) $x(t) = \cos(at)$, $y(t) = \sin(at)$, $0 \leq t \leq 2\pi$ for different values for the constants a and b. What patterns can you observe?
(4) $x(t) = t\cos t$, $y(t) = t\sin t$, $0 \leq t \leq 6\pi$.
(5) $x(t) = f(t)\cos t$, $y(t) = f(t)\sin t$, for different choices for the function $f(t)$. What do you observe about the curves you can produce?

The following questions ask you to think about when a parametric curve is the graph of a function. By that I mean a graph of the form $y = f(x)$. (If you want to do graphs $f(y) = x$ that's the same idea but swapping the variables, so let's just fix x to be the independent variable.)

(1) Consider the parametric curve given by $x(t) = t^3 + 1$, $y(t) = t^2 - t$, $-\infty < t < \infty$. Determine an equation for this curve in the form $y = f(x)$.
(2) Suppose a parametric curve is given by $x(t) = t$, $y(t) = f(t)$ for some function $f(t)$. Must this be the graph of a function? If so, which function?
(3) Suppose a parametric curve is given by $x(t) = t + b$, $y(t) = f(t)$ for some function $f(t)$ and constant b. Must this be the graph of a function? If so, which function?
(4) Suppose a parametric curve is given by $x(t) = at + b$, $y(t) = f(t)$ for some function $f(t)$ and constants a and b. Must this be the graph of a function? If so, which function?
(5) Suppose a parametric curve is given by $x(t) = e^t$, $y(t) = f(t)$ for some function $f(t)$ and constants a and b. Must this be the graph of a function? If so, which function?
(6) Can you come up with a condition which ensures $x(t) = f(t)$, $y(t) = g(t)$ gives the graph of some function? How general a statement can you make? Is your condition a *necessary condition*? (Meaning that your condition has to be satisfied if the parametric curve is the graph of a function.)

[1] If you use `desmos.com` you can graph a parametric curve by typing in something like `(cos(t),sin(t))`. If you use a graphing calculator, it probably has a mode for parametric equations but I can't tell you how to find it.