Math 02: Quiz 3

12 April 2024

Name: Answer Key

This is the third quiz. There are 10 questions. Each is worth 10 points, for a total of 100.

At the end of the quiz are 2 make-up questions for quiz 2 and 1 make up question for quiz 1. You do not have to do them if you are happy with your earlier quiz grades. If you do any make-up questions, I'll grade them and use them to replace your lowest-scored questions from the corresponding quiz.

Carefully read each question and understand what is being asked before you start to solve the problem. Please show your work in an orderly fashion, and circle or mark in some way your final answers.

No calculators nor other electronic devices are allowed.

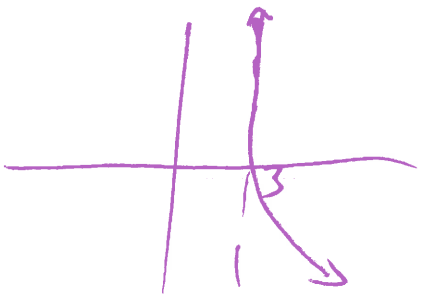
1		6	
2		7	
3		8	
4		9	
5		10	

1. Sketch a graph of $y = 2^{-x} - 4$, identifying the asymptote. What are the domain and range?

dom: all real #s

ran: $y > -4$

2. Sketch a graph of $y = -\ln(x - 3) + 1$, identifying the asymptote. What are the domain and range?



dom: $x > 3$

ran: all real #s

3. Rewrite the following equation into logarithm form:

$$4^{2x-1} = 3y$$

$2x-1 = \log_4(3y)$

4. Fully simplify the following expression, so that the input to each logarithm is as simple as possible:

$$\log_2(2^3 a x^4 / \sqrt{z})$$

$\log_2(2^3) + \log_2 a + \log_2(x^4) - \log_2(\sqrt{z})$

$= 3 + \log_2 a + 4\log_2 x - \frac{1}{2}\log_2 z$

5. Solve

$$10^{2x-4} = 1,000,000.$$

$2x-4 = 6$

$x = 5$

6. Solve

$$\log_2(2x+4) = \log_2(1-x).$$

$2x+4 = 1-x$

$x = -1$

7. Solve

$$e^{1-x} = 2.$$

$1-x = \ln 2$

$x = 1 - \ln 2$

8. Solve

$$2^{x+1} = 3^{-x}$$

$x+1 = \log_2(3^{-x})$

$= -x \log_2 3$

$x + x\log_2 3 = -1$

$x(1+\log_2 3) = -1$

$x = -\dfrac{1}{1+\log_2 3}$

9. Find the domain of

$$y = x + \log(20 - 3x).$$

dom: $20 - 3x > 0$

$-3x > -20$

$x < \frac{20}{3}$

10. The following equation describes a population of frogs over time:

$$F(t) = 1000 \times \left(\frac{9}{10}\right)^t,$$

where $F(t)$ is the number of frogs at t weeks after the state date. Determine:

- The initial population; 1 000
- Whether this is exponential growth or exponential decay; Decay
- The rate of growth/decay; and $0.1 = \frac{1}{10} = 10\%$
- The population after 2 weeks.

$F(2) = 1000 \cdot \left(\frac{9}{10}\right)^2 = 1000 \cdot \frac{81}{100} = 810$

5. **Extra Credit (up to +5):** A horizontal shift on an exponential is equivalent to a different transformation. What transformation is it equivalent to? Explain why.

$b^{x+A} = b^A \cdot b^x$. So a shift left by A is a vertical stretch by b^A, and a shift right by A is a vertical compression of $\frac{1}{b^A}$

Make-up question for quiz 1

1	

1. Complete the square to solve the following equation. You will not get credit if you use a different method.

$$x^2 - 4x = 5$$

$x^2 - 4x + 4 = 5 + 4$

$(x-2)^2 = 9$

$x - 2 = \pm 3$

$x = 2 \pm 3$

$x = -1, 5$

Make-up questions for quiz 2

1		2	

1. Fully simplify the following expression. Your answer should have no fractional exponents.

$$\frac{(2x^3)^2 y^{1/3}}{x y^{2/3}}$$

$$\frac{2^2 x^6}{x \cdot y^{\frac{2}{3}}} = \frac{4x^5}{\sqrt[3]{y}}$$

2. Find the inverse of $f(x) = \sqrt{x-1} + 2$. What are the domain and range of $f^{-1}(x)$?

$x = \sqrt{y-1} + 2$

$x - 2 = \sqrt{y-1}$

$(x-2)^2 = y - 1$

$1 + (x-2)^2 = y$

$f^{-1}(x) = 1 + (x-2)^2$

$\text{dom } f^{-1} = \text{ran } f \quad x \geq 2$

$\text{ran } f^{-1} = \text{dom } f \quad y \geq 1$

(Extra space. Label which question the work is for.)