Math 02: Quiz 4

3 May 2024

Name: Answer Key

This is the fourth quiz. There are 10 questions. Each is worth 10 points, for a total of 100.

At the end of the quiz are make-up questions for quiz 2 and quiz 3. You may do them to earn back points on a previous quiz.

Carefully read each question and understand what is being asked before you start to solve the problem. Please show your work in an orderly fashion, and circle or mark in some way your final answers.

No calculators nor other electronic devices are allowed.

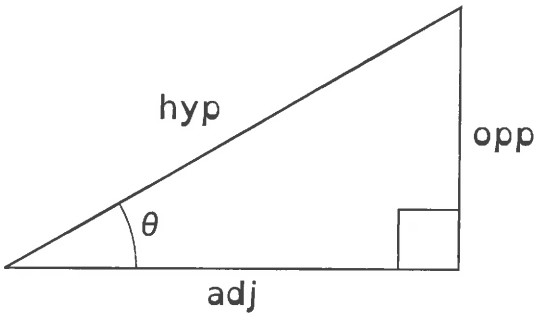
1		6	
2		7	
3		8	
4		9	
5		10	

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\text{Period} = \frac{2\pi}{B}$$

1. A right triangle with angle θ has the following sides: hyp for hypotenuse, opp for the opposite side, and adj for the adjacent side. Give all six trig functions of θ in terms of ratios of these side lengths.



$\sin\theta =$ opp/hyp

$\cos\theta =$ adj/hyp

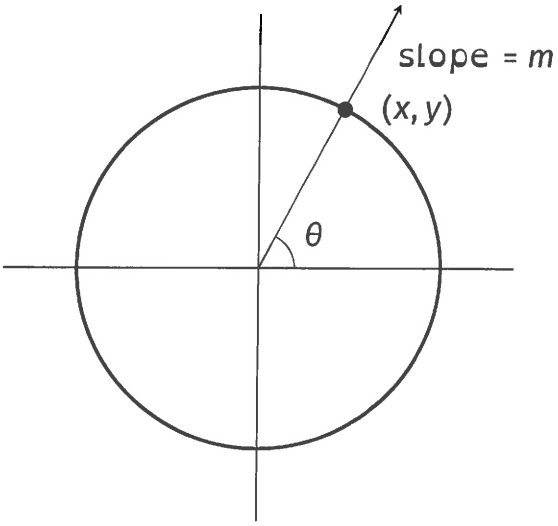
$\tan\theta =$ opp/adj

$\csc\theta =$ hyp/opp

$\sec\theta =$ hyp/adj

$\cot\theta =$ adj/opp

2. The ray coming out of the origin at angle θ intersects the unit circle at a point (x, y) and has a slope m, as in the following diagram. Give the six trig functions of θ in terms of x, y, and m. *[Note: There's multiple correct answers for some of these, but I'm only asking you to give one.]*



$\sin\theta =$ y

$\cos\theta =$ x

$\tan\theta =$ m or $\frac{y}{x}$

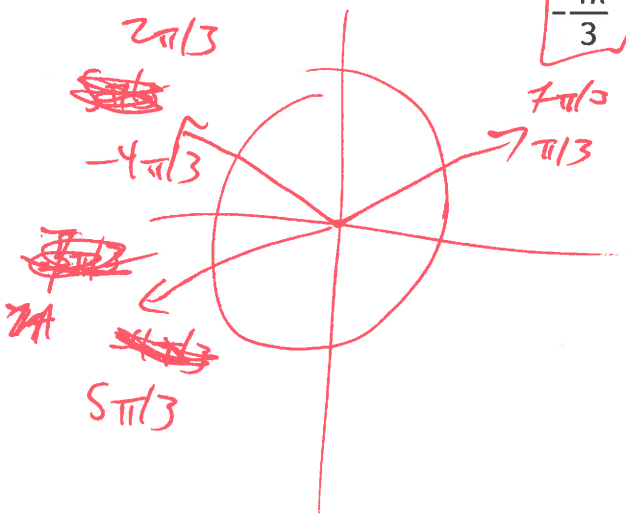
$\csc\theta =$ $\frac{1}{y}$

$\sec\theta =$ $\frac{1}{x}$

$\cot\theta =$ $\frac{1}{m}$ or $\frac{x}{y}$

3. For the following five angles, determine which of them are coterminal to each other:

$-\frac{4\pi}{3}$ $\frac{\pi}{3}$ $\frac{2\pi}{3}$ $\frac{5\pi}{3}$ $\frac{7\pi}{3}$



$-\frac{4\pi}{3} + 2\pi = \frac{2\pi}{3}$

$\frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$

$\frac{5\pi}{3}$ only angle in Q3

4. Find the reference angle of $\frac{11\pi}{7}$.

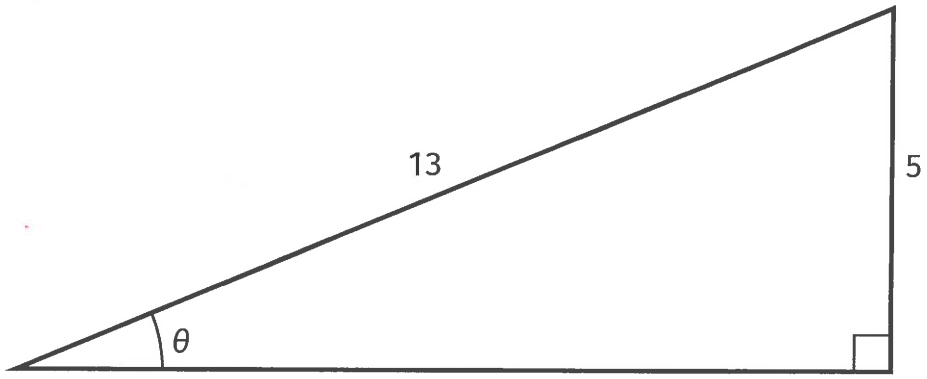
ref angle = $2\pi - \frac{11\pi}{7} = \frac{3\pi}{7}$

5. Convert the angle 40° to radians. Use this to determine whether 40° or $\frac{2\pi}{9}$ is the larger angle.

$40 \cdot \frac{\pi}{180} = \frac{4}{18}\pi = \frac{2\pi}{9}$

Same angle

6. You know two sides of a right triangle, as in the following diagram. Use this information to determine $\sin\theta$, $\cos\theta$, and $\tan\theta$.



$x = 12$

$x^2 + 5^2 = 13^2 \Rightarrow x^2 = 169 - 25 = 144$

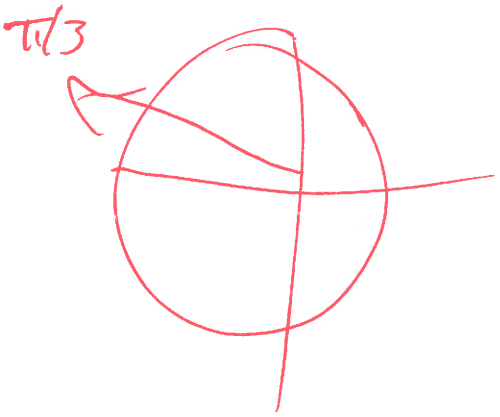
$x = 12$

$\sin\theta = 5/13$

$\cos\theta = 12/13$

$\tan\theta = 5/12$

7. You are told that $\sin(2\pi/3) = \frac{\sqrt{3}}{2}$. Use this information to determine $\cos(2\pi/3)$ and $\tan(2\pi/3)$.



$$\sin^2\left(\frac{2\pi}{3}\right) + \cos^2\left(\frac{2\pi}{3}\right) = 1$$

$$\cos(2\pi/3) = -\sqrt{1 - \sin^2\left(\frac{2\pi}{3}\right)}$$

$$= -\sqrt{1 - \frac{3}{4}} = -\sqrt{\frac{1}{4}}$$

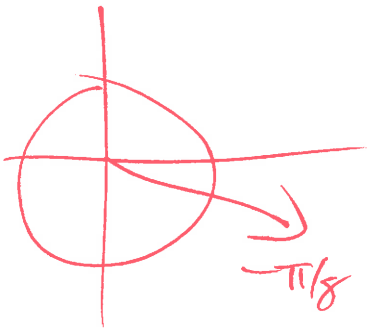
$$= -1/2$$

$$\tan(2\pi/3) = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$$

8. You are told that

$$\sin(\pi/8) = \frac{\sqrt{2-\sqrt{2}}}{2} \quad \text{and} \quad \cos(\pi/8) = \frac{\sqrt{2+\sqrt{2}}}{2}.$$

Use this information to determine $\sin(-\pi/8)$ and $\cos(-\pi/8)$.



$$\sin(-\pi/8) = -\frac{\sqrt{2-\sqrt{2}}}{2}$$

$$\cos(-\pi/8) = +\frac{\sqrt{2-\sqrt{2}}}{2}$$

9. Consider the function

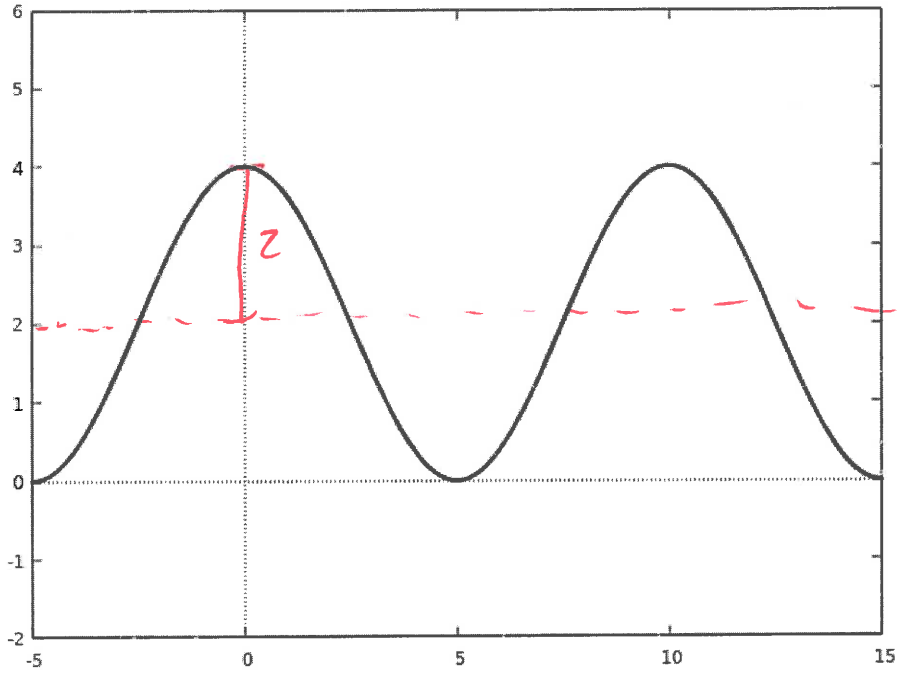
$$y = -3\sin(2x).$$

Determine the amplitude and period. Sketch a graph. Your graph should show one full period and you should label the axes so that the amplitude and period can be determined by looking at your labels.

Ampl = 3

Period = $2\pi/2 = \pi$

10. A wave oscillates between a maximum of $y = 4$ and a minimum of $y = 0$ with a period of 10, as in the graph below. Write a formula which describes the wave.



Center line $y=2$ $\quad B = \frac{2\pi}{10} = \frac{\pi}{5}$

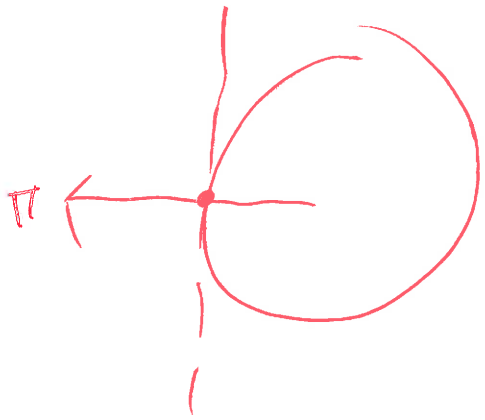
amplitude $= 2$

$$y = 2\cos\left(\frac{\pi}{5}t\right) + 2$$

11. Extra Credit (up to +5): Find all solutions to the equation

$$\cos(x) + 1 = 0.$$

$\cos x = -1$



Solutions are $x = \pi$ & all coterminal angles:

$x = \pi + 2\pi k$, where k is any integer

Make-up question for quiz 2

1	

1. Fully simplify the following expression. Your answer should have no negative nor fractional exponents.

$$\frac{(xy^{1/2})^{-1}}{2^{-2}x^{1/2}}$$

$$\frac{2^2}{x^{1/2}y^{1/2}} = \frac{4}{x^{1/2}x^{3/2}} = \frac{4}{\sqrt{y}\cdot\sqrt{x^3}}$$

$$= \frac{4}{\sqrt{x^3 y}}$$

Make-up questions for quiz 3

1	

1. Fully simplify the following expression. Each input to a logarithm should be as simple as possible.

$$\ln\left(\frac{x^2 \sin x}{\sqrt{y}}\right)$$

$$\ln(x^2) + \ln(\sin x) - \ln(\sqrt{y})$$

$$= 2\ln x + \ln(\sin x) - \frac{1}{2}\ln(y)$$

(Extra space. Label which question the work is for.)