MATH 210: 11-11 WORKSHEET
4.5 AREA BETWEEN CURVES

You can find the area between curves by integrating. To find the area of the region bounded above by the curve $y = t(x)$, below by the curve $y = b(x)$, to the left by the vertical line $x = \ell$, and to the right by the vertical line $x = r$, you compute

$$\int_{\ell}^{r} t(x) - b(x)\,\mathrm{d}x.$$

If the left and right boundaries are curves $x = \ell(y)$ and $x = r(y)$ and the top and bottom are horizontal lines $y = t$ and $y = b$, then you can find the area by integrating with respect to y:

$$\int_{b}^{t} r(y) - \ell(y)\,\mathrm{d}y.$$

If both the top/bottom and left/right boundaries are curves, you might be able to find the area by the method by subdividing your region into smaller regions bounded by lines. But in general it takes more advanced techniques in this case.

Use integration to calculate the area of the following regions. For each, you will probably find it easiest to start by drawing a picture.

(1) The rectangle with vertices $(2, 1)$, $(9, 1)$, $(9, 7)$, and $(2, 7)$. Solve this one using both an integral in x and an integral in y.
(2) The region bounded by the curves $y = x^2 + 1$, $y = x^2 - 1$, and the lines $x = -2$, $x = 2$.
(3) The bounded region enclosed by the curves $y = 4 - x^2$ and $y = x^2 - 4x + 4$. [Hint: first you need to find where the curves intersect.]
(4) The bounded region enclosed by the curves $y = x^3$ and $y = x^4$. [Same hint.]
(5) The bounded region enclosed by the curves $x = y^2$ and $x = 6 - y$. [Same hint, but your integration will be with respect to y.]
(6) The triangle with vertices $(0, 0)$, $(4, 0)$, and $(0, 4)$.
(7) The triangle with vertices $(0, 0)$, $(4, 2)$, and $(2, 4)$.
(8) The bounded region enclosed by the curves $y = x^2$ and $x = y^2$.