

MATH 210: STUDY GUIDE FOR MIDTERM 1

Here are the big things you should know for the exam.

- How to do implicit differentiation.
- How to do logarithmic differentiation.
- How to use the inverse function rule to compute derivatives.
- How to use the rules for derivatives to compute derivatives.
- How to derive the rules for derivatives, whether from the definition of the derivative or from other rules.
- How to find the equation for a tangent line.
- How to calculate the standard part of an expression involving infinitesimal and infinite numbers.
- How to determine whether an expression gives an infinitesimal, finite but not infinitesimal, or infinite number.

For the exam you will get a formula sheet with some but not all of the rules we have learned. I will post the formula sheet once it is finalized.

Here's some sample problems to practice for the exam.

- (1) Write an equation for the line tangent to the curve $e^{xy} - e^2y = 0$ at the point $(2, 1)$.
- (2) What are the slopes of the ellipse $16x^2 + 9y^2 = 25$ at the points $(\pm 1, \pm 1)$?
- (3) Differentiate $a(x) = x^{\sin x}$.
- (4) Use logarithmic differentiation to differentiate $b(x) = x^3 e^{2x} \cos x$.
- (5) Use logarithmic differentiation, the rule for the derivative of $\ln x$, and the chain rule to derive the quotient rule.
- (6) Use the other rules for the derivative to derive the rules for $\tan x$, $\cot x$, $\sec x$, and $\csc x$.
- (7) Use the other rules for the derivative to derive the rules for b^x , $\ln x$, and $\log_b x$.
- (8) Use the definition of the derivative in terms of standard systems to compute the derivative of $x^2 - x$.
- (9) Use the definition of the derivative in terms of standard systems to compute the derivative of $1/\sqrt{x}$.
- (10) Differentiate $a(x) = e^x \cos x$. What is $a'(0)$?
- (11) Find the first and second derivatives of $b(t) = 5000 + t - 10t^5$.
- (12) Differentiate $c(x) = \sqrt{1 + \ln x}$. What is $c'(1)$? Give an equation for the line tangent to $y = c(x)$ at $x = 1$.
- (13) Differentiate $d(x) = \arcsin(\tan(2x))$.
- (14) Differentiate $f(x) = \frac{e^{x^2+e}}{x^2+e}$.
- (15) Differentiate $g(x) = \arctan(x + \csc x)$.

For the following problems, ε and Δx are nonzero infinitesimals, H is positive and infinite, and x is a real number.

- (16) What is $\text{st}(5x + x\Delta x - 3\Delta x^2)$?

(17) What is

$$\text{st} \left(\frac{H^2 - H^4}{2H^4 + 100} \right)?$$

(18) What is $\text{st}(\sqrt{H+1} - \sqrt{H-1})$?

(19) Determine whether the following is infinitesimal, finite but non-infinitesimal, or infinite:

$$\frac{\varepsilon^2 + 2\varepsilon + 1}{3\varepsilon^3}.$$

(20) Determine whether the following is infinitesimal, finite but non-infinitesimal, or infinite:

$$\frac{H^2 - 1000}{3H^3 + 1}.$$

(21) Determine whether the following is infinitesimal, finite but non-infinitesimal, or infinite:

$$\frac{3}{\varepsilon} + \frac{3\varepsilon}{\varepsilon} + \frac{3\varepsilon^2}{\varepsilon}$$