MATH 210: STUDY GUIDE FOR MIDTERM 2

Here are the big things you should know for the exam.

- How to use differentiation to solve "related rates" problems.
- How to compute limits.
- How to determine where a function is continuous.
- How to find global maximums and minimums.
- How to determine where a function is increasing, decreasing, concave up, or concave down, and how to determine the locations of local minimums, local maximums, and inflection points.
- How to set up Riemann sums.
- How to calculate definite and indefinite integrals using the atomic rules for derivatives.
- How to calculate integrals using integration by substitution.

The exam consists of 8 questions. For all the types of problems, you should be able to do the calculation all the way through to the final answer. However, due to time constraints for some questions I will only ask you to do part of the setup. Carefully read each question before you start working, and take care that you don't spend time doing calculations you weren't asked to perform.

Here's some sample problems to practice for the exam.

(1) An isosceles right triangle's area is decreasing at a constant rate of 2 square feet per minute. How quickly is the length of the hypotenuse changing when it is 10 feet long?

(2) You have successfully leveraged your undergraduate degree in pure mathematics to get a job as a barista. As part of this job you are preparing a pour-over coffee for a customer, and your mind wanders back to the calculus 1 class you took. The filter you are using is conical, with a height of 2 inches and a diameter of 2 inches. You pour hot water into the filter at a rate of 1 cubic inch per second. At what rate is the depth of water in the filter increasing when you've filled it to its maximum height? [Hint: The formula for the volume of a cone is $V = \frac{\pi}{3}r^2h$.]

(3) Compute the limit
$$\lim_{x\to 2} \frac{x^2-1}{2x+1}.$$

(4) Compute the limit
$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}.$$

(5) Compute the limit
$$\lim_{x\to 0} \frac{x^4-x}{2x^2}$$

(6) On what domain is $a(x) = 3x^3+e^x$ continuous? Give your answer in interval notation.

(7) On what domain is $b(x) = \sqrt{4-3x}$ continuous? Give your answer in interval notation.

(8) On what domain is $c(x) = \frac{2x^2+x}{x^3-x}$ continuous? Give your answer in interval notation.

(9) Consider the function $d(x) = x^2 e^{-x}$ defined on the interval $-3 \leq x \leq 3$. Find the global maximum and minimum of $d(x)$ on this domain.

(10) Consider $f(x) = \sqrt[3]{x^2 + 3x + 2}$. Calculate $f'(x)$, create a sign diagram for $f'(x)$, and use this to determine where $f(x)$ is increasing or decreasing (give your answer in interval notation) and where $f(x)$ has a local maximum or local minimum.

(11) Consider $g(x) = \ln(x^2 - 4x + 6)$. Calculate $g'(x)$ and $g''(x)$. Create a sign diagram for $g''(x)$, and use this to determine where $g(x)$ is concave up or concave down (give your answer in interval notation) and where $g(x)$ has an inflection point.

(12) You are building a rectangular pasture for your goats. Your goats don't all get along, so you need the pasture to be subdivided into three regions. For simplicity, you decide to split the rectangle into three equally sized subdivisions by putting two fences down the middle. If you need to enclose a total area of 600 square feet, what is the shortest quantity of fencing you need, and what dimensions will give you that area?

(13) Find the numbers x and y which maximize the quantity $xy + y$, subject to the constraint that the numbers sum to 100 and are both nonnegative.

(14) Consider the integral

$$\int_1^3 x^3 - \frac{1}{x}\,\mathrm{d}x.$$

Set up the Riemann sum with $n = 2$ rectangles which approximates the integral, then calculate the value of the Riemann sum.

(15) Consider the integral

$$\int_0^2 4x^2 - 1.$$

Set up the Riemann sum with $n = 4$ rectangles which approximates the integral. Then write out the four terms in the sum. Do not calculate the value of the sum, but what you write out should be in a form that someone with no knowledge of calculus nor sigma notation could use to compute the value of the sum.

(16) Consider the integral

$$\int_{10}^{20} \frac{x+1}{x^2}\,\mathrm{d}x.$$

Set up the Riemann sum with $n = 5$ rectangles which approximates the integral. Then write out the five terms in the sum. Do not calculate the value of the sum, but what you write out should be in a form that someone with no knowledge of calculus nor sigma notation could use to compute the value of the sum.

(17) Integrate

$$\int e^x + x^e\,\mathrm{d}x.$$

(18) Evaluate the definite integral

$$\int_1^4 \sqrt{x}\,\mathrm{d}x.$$

(19) Evaluate the indefinite integral

$$\int \pi \sec^2 x\,\mathrm{d}x.$$

(20) Integrate

$$\int x^2 e^{x^3-4}\,\mathrm{d}x.$$

(21) Integrate

$$\int_0^1 3x^2(x^3-1)^3\,\mathrm{d}x.$$

(22) Integrate

$$\int \cos x \sin^4 x\,\mathrm{d}x.$$

(23) Integrate

$$\int_0^{\pi/2} \cos x \sin^4 x\,\mathrm{d}x.$$