MATH 217M
TRANSLATING BETWEEN NATURAL LANGUAGE AND LOGIC

A useful skill in mathematics is being able to analyze the logical structure of a mathematical statement. Another is being able to rewrite the negation of a statement using DeMorgan's laws and the equivalence $\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$.

Here is some practice with that.

(1) Write the principle of common induction (for any set X of natural numbers, if $0 \in X$ and $n \in X$ implies $n+1 \in X$ for every $n \in \mathbb{N}$, then $X = \mathbb{N}$) in logical form.

(2) In algebra, a group $(G, \star)$ is *abelian* if its operation $\star$ satisfies the property that $a \star b = b \star a$ for every $a, b \in G$.
- Write the definition of $(G, \star)$ being abelian in logical form.
- Negate this definition, and then rewrite the negation to push the $\neg$ as far inward as possible.

(3) An important notion in real analysis is the *supremum* or *least upper bound* of a set. Namely, an upper bound of a set $X \subseteq \mathbb{R}$ is a number b so that $x \leq b$ for every $x \in X$. The supremum of X, if it exists, is the least number a which is an upper bound for X. We write $\sup X = a$.
- Write the definition of $\sup X = a$ in logical form.
- Negate this definition, and then rewrite the negation to push the $\neg$ as far inward as possible.

(4) The concept of a limit is formalized by the ε-δ definition: $\lim_{x \to y} f(x) = L$ means that for every $\varepsilon > 0$ there is $\delta > 0$ so that if the distance from $x \neq y$ to y is $< \delta$ then the distance from $f(x)$ to L is $< \varepsilon$.[1]
- Write the ε-δ definition of $\lim_{x \to y} f(x) = L$ in logical form.
- Negate this definition, and then rewrite the negation to push the $\neg$ as far inward as possible.

(5) Recall that a real function f is continuous at y if $\lim_{x \to y} f(x) = f(y)$ and f is continuous on an interval I if f is continuous at y for every $y \in I$.

Using the ε-δ definition of a limit, write "f is continous on I" in logical form.

(6) A real function f is *uniformly continuous* on an interval I if it is continuous on I with the same δ working for all points in I. That is, f is uniformly continuous on I if for every $\varepsilon > 0$ there is $\delta > 0$ so that for every $x, y \in I$ if the distance from x to y is $< \delta$ then the distance from $f(x)$ to $f(y)$ is $< \varepsilon$.

Write "f is uniformly continuous on I" in logical form. Compare this to what you wrote for continuity. How do the two differ?

[1]Recall that the distance between two numbers x and y is given by the absolute value of their difference $|x - y|$.