MATH 218M
DEDUCTION RULES

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A *deduction rule* has the following form

$$\frac{A \vdash \varphi}{B \vdash \psi}.$$

Each *sequent* $A \vdash \varphi$ should be read "given the context A you can explain φ". The rule as a whole should be read: "if given the context A you can explain φ, then given the context B you can explain ψ". Some rules have multiple sequents on the top.

A *derivation* or *deduction* is a tree formed out of deduction rules, with an antecedent of one rule being a consequent of another. The very bottom of the tree, its *root*, is the sequent which the tree is a derivation of. To be a valid derivation the tree should grow upward to stop at rules with an empty antecedent, i.e. with the assumption rules (Assum).

Deduction rules for intuitionistic logic

$$\frac{A \vdash \varphi \qquad A \vdash \psi}{A \vdash \varphi \wedge \psi} \quad (\wedge\text{-Intro})$$

$$\frac{A \vdash \varphi \wedge \psi}{A \vdash \varphi} \quad (\wedge\text{-Elim})$$

$$\frac{A \vdash \varphi \wedge \psi}{A \vdash \psi} \quad (\wedge\text{-Elim})$$

$$\frac{A, \varphi \vdash \psi}{A \vdash \varphi \Rightarrow \psi} \quad (\Rightarrow\text{-Intro})$$

$$\frac{A \vdash \varphi \Rightarrow \psi \qquad A \vdash \varphi}{A \vdash \psi} \quad (\Rightarrow\text{-Elim})$$

$$\frac{A \vdash \varphi}{A \vdash \varphi \vee \psi} \quad (\vee\text{-Intro})$$

$$\frac{A \vdash \psi}{A \vdash \varphi \vee \psi} \quad (\vee\text{-Intro})$$

$$\frac{A \vdash \varphi \Rightarrow \theta \qquad A \vdash \psi \Rightarrow \theta \qquad A \vdash \varphi \vee \psi}{A \vdash \theta} \quad (\vee\text{-Elim})$$

$$\frac{A, \varphi \vdash 0}{A \vdash \neg\varphi} \quad (\neg\text{-Intro})$$

$$\frac{A \vdash \varphi \qquad A \vdash \neg\varphi}{A \vdash 0} \quad (\neg\text{-Elim})$$

$$\frac{A \vdash 0}{A \vdash \varphi} \quad (\text{Expl})$$

$$\frac{}{A, \varphi \vdash \varphi} \quad (\text{Assum})$$

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